Thinking Inside a Box

Hintikka semantics for *confident* reports

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Sinn und Bedeutung 28 – September 7, 2023

Three principles

- INTERMEDIACY
  be sure $\not\dashv\models$ *be confident* $\not\dashv\models$ *think(/believe)*

- COMMITMENT
  A is confident to some degree that p if and only if A thinks that p.

- CLOSURE
  If A is just as confident that p as they are that q, then A is just as confident that p *and* q as they are that p.

Big picture

- At least in philosophy, degrees of confidence are usually assumed to correspond to levels of subjective probability.
- INTERMEDIACY implies that the degree of confidence corresponding to the positive form *confident* isn't maximal.
- **Problem**: non-maximal levels of subjective probability don't obey COMMITMENT or CLOSURE.
- **Solution**: Hintikka semantics.
  - A is confident to at least degree d that p if and only if p is true in all of A's confident-to-at-least-degree-d worlds.
  - This entails CLOSURE.
  - And it can be combined with COMMITMENT in such a way that subjective probabilities are a necessary (but not sufficient) condition for having the corresponding degrees of confidence.

Motivating INTERMEDIACY

1. She thinks that she got an A, but isn't confident that she did.
2. # She's confident that she got an A, but it's unclear whether she thinks that she did.
3. She thinks that she got an A, and is confident that she passed.
4. She's confident that she got an A, but isn't sure that she did.
5. # She's sure that she got an A, but isn't confident that she did.
6. She's confident that she got an A, and is sure(/even more confident) that she passed.

Motivating COMMITMENT

- Suppose A thinks that p, and q is an alternative to p. It sounds fine to ask *how confident* A is that p, and fine to ask *how likely* A thinks it is that q, but it's not felicitous to ask *how confident* A is that q.

  Context: Petra and Quinn have entered an upcoming race.

7. Alice: Carl thinks that Petra will win the race.
8. Bob: How confident is he that Petra will win?
9. Bob: How likely does he think it is that Quinn will win?
10. #Bob: How confident is he that Quinn will win?

The Simple View

- Being *confident* that p to degree d just is thinking that p while assigning p subjective probability d.
- Being *confident* is having a degree of confidence above a non-maximal contextually determined threshold.

A problem for the Simple View

Context: I'm not sure which of Rock, Paper, Scissors would win.

11. I think that Rock would win, and I'm even more confident that either Rock or Paper would win.
    - $\rightsquigarrow$ This sounds non-redundant, and gives rise to the inference that I find it more plausible that Paper would win than that Scissors would win.
    - $\rightsquigarrow$ The Simple View does not predict this; nor does it predict the infelicity of the continuation:
12. # And I'm also even more confident that either Rock or Scissors would win.

Motivating CLOSURE

13. He thinks it's pretty likely that Dan arrived and thinks it's pretty likely that Eli arrived. But he thinks it's less likely that they both arrived, since maybe only one of them did.
14. ??He's pretty confident that Dan arrived and pretty confident that Eli arrived. But he's less confident that they both arrived, since maybe only one of them did.
15. #She's confident that Dan left and she's confident that Eli left. But she's not confident that they both left.

The 13/14 contrast shows that violations of CLOSURE are marked even when violations of the parallel principle about subjective probability are not. The infelicity in 15 would be explained by CLOSURE holding for the degree of confidence that is the threshold for the positive form, which given INTERMEDIACY, is presumably different non-maximal degrees in different contexts.

Hintikka semantics for gradable *confidence* reports

For all $d \in [0,1]$ and agents A, we have an accessibility relation C_A^d that maps each world w to the set $C_A^d(w)$ of worlds compatible with everything that, in w, A is confident in to at least degree d.

Definition

A's *degree of confidence* that p in w – $conf(A, p, w)$, for short – is the maximal d such that $C_A^d(w) \subseteq p$. If there is no such d then A has no degree of confidence that p in w.

$$[\![A \text{ is just as confident that } \varphi \text{ as that } \psi]\!](w) = 1 \text{ iff}$$
$$conf([\![A]\!], [\![\varphi]\!], w) = conf([\![A]\!], [\![\psi]\!], w)$$

Proposition

CLOSURE is valid.

Defining degrees of confidence

We have three ingredients:[1]

- D (*doxastic accessibility*), which interprets *think* as usual: $[\![A \text{ thinks that } \varphi]\!](w) = 1$ iff $D_{[\![A]\!]}(w) \subseteq [\![\varphi]\!]$
- Pr (the *subjective probability* function), where $Pr_{A,w}$ is a probability distribution over W;
- $\succeq$ (the *plausibility order*), where $\succeq_{A,w}$ is a (well-founded) total preorder on W.

Definition

$C_A^d(w) = D_A(w) \cup \bigcap\{p : Pr_{A,w}(p) \geqslant d$, and $v \in p$ for all $v \succeq_{A,w} u, u \in p\}$.

Proposition

COMMITMENT is valid.

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[1] These probably aren't independent: Holguín [2022] argues that comparative plausibility (i) constrains doxastic accessibility and (ii) can be defined in terms of subjective probabilities relative to a contextually determined question.

Unpacking the definition

- A special case:
  - A is *reasonably opinionated* $:= D_A(w) = \{v : v \geq u \text{ for all } u\}$. I.e., the doxastic possibilities are all and only the most plausible possibilities.
  - If A is reasonably opinionated, then A's degree of confidence in p is the probability of largest non-empty p-entailing set that includes every world at least a plausible as any it contains.
- We can think of $\geq$ as a determining a nested system of spheres (as in Lewis's [1973] semantics for conditionals).
- The definition then says that A is confident to at least degree d that p iff A both think that p and p is true throughout some sphere to which A assigns at least d subjective probability.
- So being confident to at least degree d that p entails having at least d subjective probability in p, and also entails being confident to degree at least d' that p for all $d' < d$.

Positive forms

These receive standard degree-theoretic truth conditions for gradable adjectives via contextually determined thresholds.

$[\![A \text{ is confident that } \varphi]\!]^c(w) = 1$ iff $conf([\![A]\!]^c, [\![\varphi]\!]^c, w) \geqslant \theta_c^{\text{conf}}$

- equivalently, $C_{[\![A]\!]^c}^{\theta_c^{\text{conf}}}(w) \subseteq [\![\varphi]\!]^c$

$[\![A \text{ is sure that } \varphi]\!]^c(w) = 1$ iff $conf([\![A]\!]^c, [\![\varphi]\!]^c, w) \geqslant \theta_c^{\text{sure}}$

- equivalently, $C_{[\![A]\!]^c}^{\theta_c^{\text{sure}}}(w) \subseteq [\![\varphi]\!]^c$

Securing INTERMEDIACY

- We require that $\theta_c^{\text{conf}} \leqslant \theta_c^{\text{sure}}$ and allow contexts where $\theta_c^{\text{conf}} < \theta_c^{\text{sure}}$.
- We also require $\theta_c^{\text{conf}} > .5$, to ensure that *confident* isn't 'weak' like *think(/believe)* are [Hawthorne et al., 2016]:

16. Carl thinks(/believes) that Petra will win, but thinks she is only 40% likely to win.
17. #Carl is confident Petra will win, but thinks she is only 40% likely to win.

Topic for future work: question sensitivity

- Yalcin [2018] and Holguín [2022] argue that belief is always relative to a contextually supplied question.
  - E.g., although 7 is felicitous in a context where Alice is focused on *who will win*, the opposite report *Carl thinks Petra will lose* might be felicitous in a context where she is focused on *whether Petra will win*.
- Given COMMITMENT, degrees of confidence must exhibit parallel question-sensitivity.
  - This is independently motivated by work in formal epistemology on the question-sensitivity of plausibility orders.[2]

[2] Question-sensitivity offers a natural way to derive plausibility orders from probabilities [Goodman and Salow, 2021, Dorst and Mandelkern, forthcoming]: $v \succeq^{Q}_{A,w} u$ iff $Pr_{A,w}([v]_Q) \geqslant Pr_{A,w}([u]_Q)$, where Q is a partition of W and $[w]_Q$ is its cell that contains w. Holguín [2022]'s 'cogency' generalization about belief then amounts to $D^{Q}_{A}(w)$ being nonempty and closed under $\succeq^{Q}_{A,w}$.

Topic for future work: *confident* jargon

What should we make of philosophers' practice of using *confident* to report subjective probabilities?

- It's unclear how continuous it is with ordinary usage. E.g., of the $\sim 14,300$ Google hits for *times as confident that*, only 10 remained when I excluded the word *philosophy*!

What about Cariani et al. [2021]'s leading example of comparative confidence (below)?

18. Ann is more confident that it's raining than that it's snowing.

The intended reading is that Ann's subjective probability is greater in rain than in snow. However, 18 has no true reading on the present proposal (assuming *it's raining* and *it's snowing* are contraries) – by COMMITMENT, it requires Ann to have inconsistent beliefs; but then $D_A(w) = \varnothing$ and Ann's degree of confidence in every proposition will be 1.

References

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