Analysis Advance Access published October 5, 2016

Reality is not structured

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The identity predicate can be defined using second-order quantification: $a = b =_{df} \forall F(Fa \leftrightarrow Fb)$. Less familiarly, a dyadic sentential operator analogous to the identity predicate can be defined using third-order quantification: $\varphi \equiv \psi =_{df} \forall X(X\varphi \leftrightarrow X\psi)$, where X is a variable of the same syntactic type as a monadic sentential operator. With this notion in view, it is natural to ask after general principles governing its application. More grandiosely, how fine-grained is reality?

I will argue that reality is not structured in anything like the way that the sentences we use to talk about it are structured. I do so by formulating a higher-order analogue of Russell's paradox of structured propositions. I then relate this argument to the Frege-Russell correspondence. When confronted with the alleged paradox, Frege agreed that reality was not structured, but maintained that propositions (i.e. thoughts) were structured all the same. Russell replied that his paradox showed Frege's theory of structured thoughts to be inconsistent, to which Frege replied that Russell's argument failed to heed the distinction between sense and reference. Most recent commentators have sided with Russell. In defense of Frege, I establish the consistency of one version of his rejoinder. I then consider and reject some ways of resisting the argument against a structured conception of reality. I conclude that, if propositions are structured, this is because they correspond not to distinctions in reality, but rather to ways in which those distinctions can be represented.

1. Against structure

I will work in a higher-order language with variables p, q, ... of the same syntactic type as sentences and X, Y, ... of the same syntactic type as monadic sentential operators. I will assume the following two standard principles of higher-order logic[1]:

λ-CONV
$\Phi[\varphi/p] \leftrightarrow (\lambda p\Phi)\varphi$, where φ is free for p in Φ

∃-INTRO
$\Phi[t/v] \rightarrow \exists v\Phi$, where t is free for v in Φ and has same syntactic type as v

1 As usual, $(\lambda p\Phi)$ is a monadic sentential operator in which all occurrences of p are bound, $\Phi[\varphi/p]$ abbreviates the result of replacing zero or more free occurrences of p in Φ with φ, and φ is free for p in Φ just in case, for all variables v, no free occurrence of v in φ becomes bound when φ is substituted for some free occurrence of p in Φ.

Analysis Vol 0 | Number 0 | September 2016 | pp. 1–10 doi:10.1093/analys/anw002

Let $\Phi(X)$ schematically stand for a formula in which at most the variable X is free and $p \approx q$ schematically stand for a formula in which at most the variables p and q are free; let $\Phi(O)$ stand for the result of uniformly substituting the monadic sentential operator O for all free occurrences of X in $\Phi(X)$, and $\varphi \approx \psi$ stand for the result of uniformly substituting φ and ψ, respectively for all free occurrences of p and q in $p \approx q$. Now consider the following schematic derivation. (For brevity, we omit premisses that are either theorems of classical propositional logic or axioms governing vacuous quantification and quantifier distribution.)

$$O =_{df} (\lambda p \exists X((\Phi(X) \approx p) \wedge \neg Xp))$$

(1) $\Phi(O) \approx \Phi(O)$ — assumption
(2) $((\Phi(O) \approx \Phi(O)) \wedge \neg O\Phi(O)) \to \exists X((\Phi(X) \approx \Phi(O)) \wedge \neg X\Phi(O))$ — ∃-INTRO
(3) $\exists X((\Phi(X) \approx \Phi(O)) \wedge \neg X\Phi(O)) \to O\Phi(O)$ — λ-CONV (left-to-right)
(4) $O\Phi(O)$ — 1, 2, 3
(5) $O\Phi(O) \to \exists X((\Phi(X) \approx \Phi(O)) \wedge \neg X\Phi(O))$ — λ-CONV (right-to-left)
(6) $\exists X((\Phi(X) \approx \Phi(O)) \wedge (\neg X\Phi(O) \wedge O\Phi(O)))$ — 4, 5
(7) $\exists X \exists Y((\Phi(X) \approx \Phi(Y)) \wedge (\neg X\Phi(O) \wedge Y\Phi(O)))$ — 6, ∃-INTRO
(8) $\exists X \exists Y((\Phi(X) \approx \Phi(Y)) \wedge \exists p(\neg Xp \wedge Yp))$ — 7, ∃-INTRO
(9) $(\Phi(O) \approx \Phi(O)) \to \exists X \exists Y((\Phi(X) \approx \Phi(Y)) \wedge \exists p(\neg Xp \wedge Yp))$ — 1, 8

Letting $\ulcorner \langle p \rangle \urcorner$ abbreviate $\ulcorner$the proposition that $p\urcorner$, and instantiating $\Phi(X)$ with '$\forall p(Xp \to p)$' and $p \approx q$ with '$\langle p \rangle = \langle q \rangle$' in our schematic conclusion (9) yields:

(10) $$\langle \forall p(Op \to p) \rangle = \langle \forall p(Op \to p) \rangle \to \exists X \exists Y(\langle \forall p(Xp \to p) \rangle = \langle \forall p(Yp \to p) \rangle \wedge \exists p(\neg Xp \wedge Yp))$$

The antecedent of (10) is clearly true, given the Platonist assumption that the 'the proposition that ...' construction is in good standing. So, by modus ponens, we may conclude:

(11) $\exists X \exists Y(\langle \forall p(Xp \to p) \rangle = \langle \forall p(Yp \to p) \rangle \wedge \exists p(\neg Xp \wedge Yp))$

This conclusion is a version of the Russell-Myhill antinomy.[2] It says that, for some X and Y, the proposition that $\forall p(Xp \to p)$ is identical to the proposition that $\forall p(Yp \to p)$ despite the fact that X and Y are not even co-extensive.

The mode of argument has force even for nominalists who deny that there are any such abstract objects as propositions. For suppose we instead instantiate $p \approx q$ with the formula '$\forall Z(Zp \leftrightarrow Zq)$', which makes no reference to propositions. Our schematic conclusion (9) then becomes:

(12) $$\forall Z(Z\forall p(Op \to p) \leftrightarrow Z\forall p(Op \to p)) \to \exists X \exists Y(\forall Z(Z\forall p(Xp \to p) \leftrightarrow Z\forall p(Yp \to p)) \wedge \exists p(\neg Xp \wedge Yp))$$

2 Originally from Russell (1903, Appendix B).

Since the antecedent of (12) is clearly true, by modus ponens we may conclude:

(13) $\exists X \exists Y (\forall Z (Z \forall p (Xp \rightarrow p) \leftrightarrow Z \forall p (Yp \rightarrow p)) \wedge \exists p (\neg Xp \wedge Yp))$

In other words, for some X and Y, $\forall p(Xp \rightarrow p)$ is higher-order indiscernible from $\forall p(Yp \rightarrow p)$ despite the fact that X and Y are not even co-extensive. Such indiscernibility is a higher-order analogue of identity, since it has the same logical behaviour with respect to its sentential arguments as the identity predicate has with respect to its nominal arguments: both are reflexive and (given λ-CONV and ∃-INTRO) license the intersubstitution of their flanking expressions. Using this notion of 'identification', we can formulate claims about reality's fineness of grain without appealing to a first-order ontology of propositions.[3] Our conclusion (13) therefore expresses a strong constraint on reality's fineness of grain that applies even to nominalists, provided they are willing to theorizing in higher-order terms. It implies that reality is not structured in the manner of the sentence we use to talk about it. For while the distinctness of the variables 'X' and 'Y' suffices for the non-identity of the formulas '$\forall p(Xp \rightarrow p)$' and '$\forall p(Yp \rightarrow p)$', the non-coextensiveness of conditions X and Y does not preclude the identification of $\forall p(Xp \rightarrow p)$ and $\forall p(Yp \rightarrow p)$.

Here is a different way of putting the point. Given λ-CONV, ∃-INTRO and classical logic, we must reject the schema

STRUCTURE
$\forall X \forall Y (\Phi(X) \approx \Phi(Y) \rightarrow (\Psi(X) \leftrightarrow \Psi(Y)))$, where X occurs free in $\Phi(X)$

for any reflexive condition $\approx$ (be it propositional identity, higher-order indiscernibility, or whatever) since, provided that $\Phi(O) \approx \Phi(O)$, instantiating $\Psi(X)$ with $\ulcorner X\Phi(O) \urcorner$ yields a provably false instance, whatever the condition Φ. The schema is so named because it attempts to give expression to the informal idea that, if X and Y occur at the same position in the same structure Φ, then they are the same and so are intersubstitutable in any context Ψ.[4]

2. *Frege versus Russell*

Frege accepted claim (13) about the granularity of reality while resisting the analogous claim (11) about the granularity of propositions (i.e. thoughts). According to him, the only distinctions in reality are extensional distinctions, and so reality is extremely coarse-grained; by contrast, thoughts are structured

3 Alternatively, we might introduce a primitive dyadic sentential operator analogous to the identity predicate as a formalization of a certain reading of the English construction 'For it to be the case that ... just is for it to be the case that ...'; see Rayo (2013) and Dorr (forthcoming).

4 We could instead use fourth-order quantification to define a notion of identification that takes monadic sentential operators as argument, and then replace the consequent of STRUCTURE with the identification of X and Y.

entities built out of their constituent senses. In his correspondence with Russell, Frege claimed that no paradox arose for thoughts so concieved and that Russell's argument to the contrary fallaciously conflated sense and reference.[5] Recently, a number of authors have argued that Frege was wrong. In particular, they have argued that, if thoughts are structured out of senses in the way Frege imagined them to be, then we should be able to derive a contradiction by an application of Cantorian diagonalization.[6] This is certainly true for some theories of Fregean senses. For example, Myhill (1958) famously derived a contradiction in the system in Church (1951) by combining a derivation analogous to the one above with axioms of that system inconsistent with its schematic conclusion.[7] But not all Fregean theories are doomed to such a fate. In defense of this claim, I will now describe a toy model establishing the consistency of a broadly Fregean response to the Russell-Myhill antinomy.

According to Fregeans, 'the proposition that' creates an opaque context, in the sense that the truth of an identity statement involving proper names fails to license the intersubstitution of those names in that context. Quantification into opaque contexts calls for special treatment. The simplest approach is to treat 'the proposition that' as crypto-quotational and quantification into its scope as crypto-substitutional. We recursively assign sentences truth conditions by treating them as synonymous with the result of applying the following recursive translation. First, some definitions: an occurrence of a variable v in a formula φ is *problematic* if it is free in φ and occurs under 'the proposition that'; the *problem set* of a formula is the set of variables with problematic occurrences in that formula; a *problematic* formula is one with a non-empty problem set; a *target* formula is a problematic formula having no problematic proper subformulas. The translation proceeds by prefixing every target subformula φ with $\ulcorner$for some closed expression e of type t that means $v\urcorner$ for every variable v in the problem set of φ (in some order), where t is the syntactic type of v and 'means' is a placeholder for the semantic notion appropriate to closed expressions of type t (e.g. 'denotes' in the case of names). We then replace the problematic occurrences of v with occurrences of e. Next, we replace all occurrences of $\ulcorner$the proposition that $\varphi\urcorner$ in the subformula with $\ulcorner\ulcorner\varphi\urcorner\urcorner$, rendering the subformula unproblematic, and then replace predicates of propositions with corresponding predicates of sentences, rendering it sensible. We repeat this process until the sentence is no longer problematic, and then lastly replace any remaining occurrences of $\ulcorner$the proposition that $\varphi\urcorner$ with $\ulcorner$'φ'$\urcorner$.

Let's work a simple example. Consider the problematic sentence '$\exists x(x = \text{Superman}$ and Lois believes the proposition that x flies)'. Its only

5 See Frege (1980).

6 See Rieger (2002) and Klement (2001, 2002, 2003, 2005).

7 Klement (2002) does the same for a system designed to be more faithful to Frege's own views than Church's was.

target subformula is 'Lois believes the proposition that x flies', and 'x' is the only member of its problem set. So we prefix that subformula with 'for some name n, n denotes x' (since 'x' is a nominal variable), replace the problematic occurrence of 'x' with 'n', replace 'the proposition that n flies' with '$\ulcorner n$ flies$\urcorner$', and replace 'believes' with 'believes-true', yielding the unproblematic sentence '$\exists x(x =$ Superman and for some name n, n denotes x and Lois believes-true $\ulcorner n$ flies$\urcorner$)' as its translation.

These truth conditions falsify our earlier conclusion (11) that, for some non-coextensive X and Y, the proposition that $\forall p(Xp \rightarrow p)$ is identical to the proposition that $\forall p(Yp \rightarrow p)$, since if X and Y are non-coextensive, then there are no sentential operators O and O' such that O means X, O' means Y, and $\ulcorner \forall p(Op \rightarrow p)\urcorner = \ulcorner \forall p(O'p \rightarrow p)\urcorner$—indeed, these truth conditions validate STRUCTURE for the same reason.[8] The derivation of that conclusion is blocked because $\exists$-INTRO becomes invalid in opaque contexts. For example, 'Superman = Clark and Lois does not believe the proposition that Clark flies' is true, since Lois does not believe-true 'Clark flies', but '$\exists x$(Superman $= x$ and Lois does not believe the proposition that x flies)' is false, since everything identical to Superman has a name n, namely 'Superman', such that Lois believes-true $\ulcorner n$ flies$\urcorner$. The same phenomenon arises with higher-order quantification into opaque contexts: assuming that sentential operators mean at most one thing (up to extensional equivalence), the truth conditions given above entail that either step (2) or (7) of the above derivation fails (since the truth conditions validate $\exists$-INTRO in non-opaque contexts, λ-CONV, and the classical background logic).[9]

Of course, propositions are not actually sentences. The above truth conditions therefore do not give the intended interpretation of sentences containing quantification into the scope of 'the proposition that'. But they were not intended to do so. They were merely intended to illustrate how Fregeans can insulate their 'third realm' of thoughts from Cantorian contradiction by taking opacity seriously. Moreover, following Kaplan (1968), it is straightforward to modify the translation so that the senses out of which propositions are built are not identified with linguistic expressions. We simply replace the Quinean corner quotes with Kaplanian sense quotes, replace quantification over linguistic expressions with appropriately restricted quantification over senses, and replace $\ulcorner$means $v\urcorner$ with $\ulcorner$is a sense that determines the referent $v\urcorner$. The advantage of the crude quotational semantics is that, not only does it provide a concrete model of how the failures of $\exists$-INTRO

8 I am here assuming that no monadic sentential operator both means X and means Y for non-coextensive X and Y. If this is denied, we can replace meaning with meaning* in the above translation, where the meaning* of an operator is the conjunction all of its meanings.

9 Absent substantive semantic assumptions, the truth conditions leave open which of these two instances of $\exists$-INTRO fails.

associated with broadly Fregean theories of quantifying in block the argument against structured propositions, but it also shows how such failures make room for propositions being structured in the manner of the sentences we use to express them.

3. *Interpreting higher-order quantification*

The derivation of (13) makes use of both quantification into sentence position and quantification into monadic sentential operator position. How should such quantification be understood?

On a certain Platonist interpretation, quantification into sentence position is understood as disguised first-order quantification over propositions and quantification into monadic sentential operator position is understood as first-order quantification over classes of propositions. We might then read p as ⌜p is true⌝, ⌜$X\varphi$⌝ as ⌜the proposition that φ is a member of X⌝, etc. On this interpretation, the above derivation is simply a tidy notation for the derivation Russell gave in 1903, since (13) then becomes Russell's original conclusion that there are two classes of proposition, X and Y, such that the proposition that all members of X are true is identical to the proposition that all members of Y are true despite X and Y having different members. Friends of structured propositions might then resist this conclusion by denying that there are as many classes of propositions as ∃-INTRO (so interpreted) entails. Deutsch (2014), for example, proposes denying that all propositions can be members of classes, the idea being that some propositions are about too many things to be members of any class.

Uzquiano (2015) argues that this response fails to get to the heart of the antinomy, since quantification into sentential operator position can be interpreted in terms of plural quantification over propositions rather than in terms of quantification over classes of propositions. The analogue of Deutsch's suggestion would then be to deny that every proposition is such that there are some propositions of which it is one, which has no plausibility.

Although this plural interpretation of quantification into monadic sentential operator position avoids talk of classes, it is still Platonist in that such quantification is understood in terms of quantification over propositions, understood as (presumably abstract) objects. But as I mentioned earlier, the real moral of the derivation—that reality is not structured—is not hostage to such Platonism.

Following Prior (1971) and Williamson (2003) (and, I would argue, Frege 1879), quantification into non-nominal syntactic positions can simply be understood on its own terms, without any need for nominalization or for any other scheme for translating it into English. Here is not the place to defend this claim. But as someone who believes that higher-order languages, so understood, are one of our most powerful tools for metaphysical theorizing, let me make two points about how the derivation looks from that perspective.[10]

First, on a primitivist understanding of quantification into sentential operator position, instances of ∃-INTRO involving complex expressions like O which themselves contain higher-order quantifiers are no less intuitively valid than instances involving expressions that do not contain such quantifiers. So while accepting only the latter instances of ∃-INTRO suffices to block the above derivation, such a restriction seems highly implausible.[11]

Second, a primitivist interpretation of higher-order quantification is non-Platonist but it is not *anti*-Platonist. It does not involve *denying* that there are such individuals as propositions. It simply denies that quantification into sentence position is disguised first-order quantification over propositions. Indeed, the derivation of (11) tacitly *assumes* the existence of propositions in arguing that they cannot be as fine-grained as structured accounts would have them be. Of course, someone might take that conclusion as a reason to deny that there are any propositions to begin with. But, to reiterate, even if that reaction is correct, we can still formulate questions about the granularity of reality in purely higher-order terms without recourse to first-order quantification over propositions (or facts, states of affairs, or individuals of any kind), and (13) places stringent constraints on how we answer such questions.

4. *Against deep structure*

In a recent discussion of the Russell-Myhill antinomy, Hodes (2015) argues that principles like STRUCTURE have little antecedent plausibility, since they

10 Such an interpretation of the Russell-Myhill antinomy is briefly mentioned by Klement (2014) and Uzquiano (2015).

11 Walsh (2016) explores responses to the Russell-Myhill antinomy that reject such 'impredicative' instances of ∃-intro. Although the Fregean response considered earlier also requires giving up ∃-INTRO, it did not require giving it up in non-opaque contexts. For example, unlike responses that ban impredicativity, the Fregean response validates $\ulcorner \exists p \forall X(Xp \leftrightarrow X\varphi) \urcorner$, provided X is not free in φ. Indeed, the ban on impredicativity has to be understood in a particularly draconian way in order to block the paradox: in particular, it has to deny that the higher-order realm is closed under application. This would be like denying that there is such a condition as loving John, despite there being such a person as John and such a relation as love. Informally, we need this restriction because, assuming only predicative comprehension, we can establish the existence of a property of properties of propositions that applies to all and only properties that apply to at least one proposition: in other words, we can establish that there is such a thing as the existential propositional quantifier (at least up to extensional equivalence), understood as a higher-order entity, which we can then use as a parameter in place of higher-order quantification in the above derivation; the same goes for quantification into monadic sentential operator position.

are flatly inconsistent with of STRONG λ-CONV: the result of replacing the biconditional in λ-CONV with an appropriate higher-order analogue $\approx$ of identity.[12] For that principle immediately yields $\lambda p(p \approx p)\varphi \approx \lambda p(p \approx \varphi)\varphi$, which is incompatible with STRUCTURE, since $\lambda p(p \approx p)$ unlike $\lambda p(p \approx \varphi)$ has universal extension.

I think this more direct argument against STRUCTURE is perfectly correct, although perhaps not as dialectically effective as the Russell-Myhill-style argument. For present purposes, the interesting question is whether it points to a way of reformulating the idea that reality is structured that evades both arguments.

A tempting thought is that the superficial predicative structure of our language can fail to mirror the fundamental language-like structure of reality.[13] Distinctions in reality are built quasi-syntactically out of certain fundamental ingredients. It is hard to know how to formulate this idea precisely in a higher-order language. But at least part of the idea seems to be that, just as we can ask what sentence results from uniformly replacing an expression in a given sentence by another expression of the same type, so we can ask what claim about reality results from substituting for some fundamental constituent of a given claim a different entity of the same type. And this idea turns out to be in tension with STRONG λ-CONV.

Suppose that some fundamental dyadic relation R is symmetric, in the sense that $R \approx R^*$, where R^* is the converse of R (i.e. $\lambda xyRyx$). (In fact, we need only suppose that some polyadic relation can be identified with one of its non-trivial permutations.) This seems like something we ought to allow.[14] By the analogue of Leibniz's law for $\approx$, we have $(R \approx R) \approx (R \approx R^*)$. (Let a given occurrence of $\approx$ be disambiguated by the type of its flanking expressions.) By a suitably general formulation of STRONG λ-CONV, we have $(R \approx R) \approx \lambda S(S \approx S)R$ and $(R \approx R^*) \approx \lambda S(S \approx S^*)R$. So $\lambda S(S \approx S)R$ and $\lambda S(S \approx S^*)R$ are one and the same claim—call it p. Now consider some non-symmetric relation R'. What claim about reality results from substituting R' for R in p? The question seems to be ill-posed, since both the triviality $\lambda S(S \approx S)R'$ and the falsehood $\lambda S(S \approx S^*)R'$ seem to have equal claim. If the question is indeed ill-posed, this constitutes a crucial disanalogy

12 Hodes argues that Russell is committed to STRONG λ-CONV given his views about propositional functions.

13 Compare Sider's (2011) the metaphor of the 'book of the world'.

14 Indeed, Dorr (2004) argues that all fundamental relations are symmetric with respect to all of their permutations. The proponent of fundamental structure cannot object that R must be less complex than R^*, at least if they are thinking on the model of syntactic complexity, since then Rab would be less complex than $R^{**}ab$ but STRONG λ-CONV entails that $Rab \approx R^{**}ab$.

between the structure of distinctions in reality and the syntactic structure of the sentences we use to draw those distinctions.[15]

5. Conclusion

How fine grained is reality? This is perhaps the deepest question in all of metaphysics, and higher-order languages provide the tools to precisely formulate and productively debate competing answers to it. They also afford us a better view of the question. For rather than asking about the granularity of this or that supposed realm of abstract objects, higher-order quantification allows us to ask in *unrestricted generality* about the granularity of reality itself. Less grandiosely, and more precisely, it allows us to ask after general principles governing a dyadic sentential operator $\approx$ analogous to the first-order identity predicate. Assuming λ-CONV and ∃-INTRO, we have a powerful limitative result: reality is not structured in anything like the way that the sentences we use to talk about it are. If Fregeans are right that 'the proposition that' creates an opaque context, then perhaps propositions are structured all the same. But if so, this structure reflects ways in which reality can be represented, not the structure of reality itself.[16]

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15 For a similar argument in a slightly different context, see Fine (2000, footnote 19).

16 An earlier version of this paper was presented at Washington Square Circle at NYU. Thanks to the audience for their questions, and also to Andrew Bacon, Harvey Lederman, Beau Mount, Jeff Russell and Gabriel Uzquiano for discussion. Special thanks to Cian Dorr and Peter Fritz for invaluable comments on multiple drafts of this material.

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