Matter and mereology*

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I am a material thing. But I am not the same thing as the matter out of which I am composed, since my matter, unlike me, could have existed as scattered interstellar dust. The distinction between matter and objects that are merely composed of matter is central to our ordinary conception of the material world. According to that conception, material objects have a hierarchical structure with matter at its foundation. This paper shows how matter and material constitution can be understood in terms of the part-whole relation.[1] I present a novel mereology and apply it to debates about the persistence and plenitude of material objects, and compare my view to more familiar hylomorphic ones. A formal model of the theory is given in an appendix.

1 Anti-symmetry

Consider the following principle:

> WEAK SUPPLEMENTATION: If x is a proper part of y, then y has a part that does not overlap x.

where

> x is a *proper part of* $y =_{df}$ x is a part of y and $x \neq y$.
>
> x *overlaps* $y =_{df}$ some part of x is part of y.

Everyday cases of material coincidence are counterexamples to WEAK SUPPLEMENTATION. A statue s composed of some clay c. c is distinct from s, since squashing c would destroy s but would not destroy c.[2] c is part of s, since any

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[1] This paper is concerned with the ordinary notion of part according to which material objects can have different parts at different times, which should not be confused with the notion of 'atemporal parthood' postulated by Lewis (1986) and others. See footnote 12.

[2] This claim is contentious, but a defense of it is beyond the scope of this paper.

matter is part of any material object composed of it.[3] So c is a proper part of s. Every part of s has a part of c as a part, and hence overlaps c.[4] So s and c are a counterexample to WEAK SUPPLEMENTATION.[5]

Now consider the following principle:

> ANTI-SYMMETRY: If x is part of y and y is part of x, then $x = y$.

ANTI-SYMMETRY and WEAK SUPPLEMENTATION are often treated as if they were a package deal.[6] But they aren't. Given the reflexivity and transitivity of parthood, WEAK SUPPLEMENTATION is strictly stronger than ANTI-SYMMETRY. Everyday cases of material coincidence pose no obvious challenge to ANTI-SYMMETRY. If anything, we are naturally inclined to deny that s is part of c. So we should explore the possibility of maintaining ANTI-SYMMETRY while rejecting WEAK SUPPLEMENTATION. Such views turn out to have substantial theoretical advantages over the more popular rival view that distinct coinciding material objects are parts of each other. Or so I will argue.

2 Matter

Our ordinary conception of material objects distinguishes matter, like the clay c, from objects merely composed of matter, like the statue s.[7] It is natural to articulate this conception mereologically:

> MATTER: Something is matter just in case it is a material object that does not coincide with any of its proper parts.

> MATERIALITY: Something is a material object just in case it has as a part some matter with which it coincides.

where

> *x coincides with y* $=_{df}$ x and y overlap the same things.

[3] Simons (1987) rejects this principle in order to save WEAK SUPPLEMENTATION, which he takes to be analytic.

[4] *Pace* Koslicki (2008), who argues from WEAK SUPPLEMENTATION to the claim that s has a 'formal part' that does not overlap c.

[5] Some writers prefer to reserve the term 'proper part' for *non-coincident* parts. Although nothing of substance turns this terminological decision, it is worth noting that WEAK SUPPLEMENTATION is true on this alternative reading of 'proper part'.

[6] See Thomson (1983), Hawthorne (2006a), and Hovda (2013), who reject both principles.

[7] It is somewhat contentious whether c is the matter of s, since although the matter of s would survive its constituent elementary particles being dispersed throughout the galaxy, it is not clear that c would survive such dispersal. This point does not threaten the above argument against WEAK SUPPLEMENTATION, since even those who deny that c is the matter of s should agree that it is part of s, and in any event we can run that argument with the matter of s in place of c.

But MATTER and MATERIALITY are jointly inconsistent with views according to which distinct coincident material objects are parts of each other.[8] Those who accept such views typically reject MATTER. How are they to distinguish matter from other material objects?[9]

It is often claimed that matter is distinguished from other material objects in having its parts essentially: if x is matter and y is part of x, then x cannot exist without having y as a part. But this claim must be rejected by those who think that material coincidence entails mutual parthood, since according to that view, s is part of c (since they coincide), but only contingently (since c can survive the destruction of s), and so c does not have all its parts essentially.

A more promising strategy for characterizing matter is by appeal to physics. Many physical laws seem to have restricted application to matter. Consider the law that nothing travels faster than light. Imagine an amphitheater with a performer standing in the middle. The performer points into the stands and sweeps his arm from left to right. When the members of the audience see him point to them, they raise their arms. Now consider the resulting wave: the material object that at any time has the same location as the audience members whose hands are raised at that time. The wave will travel at a speed proportional to the radius of the theater. If the theater is large enough, the wave will travel faster than light. So it is consistent with the laws of physics that a material object travels faster than light. The seemingly contrary generalization should be understood as tacitly restricted to matter.

Hawthorne (2006a) is naturally read as suggesting that those who deny ANTI-SYMMETRY try to leverage such facts to give a characterization of matter in terms of physical laws. (He makes this suggestion after canvassing a range of other proposals and finding them all wanting.) He proposes, in effect, that for a material object to be matter is for it to have some property that figures in the laws of fundamental physics. Perhaps, for example, unit negative charge is a property had only by elementary particles, although other material objects can be said to have unit negative charge in the derivative sense of having one more

[8] Suppose x and y are distinct coincident material objects. By MATERIALITY, there is some matter m with which they coincide. If coincident material objects are mutual parts, then x and y are both parts of m. Since x and y are distinct, at least one of them is a proper coincident part of m, contradicting MATTER.

Note that MATTER and MATERIALITY are compatible with the view that *some* distinct coincident material objects are parts of each other. Doepke (1982) accepts both theses but also thinks that people and their bodies are a counterexample to ANTI-SYMMETRY.

To be clear, neither MATTER or MATERIALITY are offered as *definitions*, and treating both as such would obviously be circular. I take the notions of matter and material object to be clear enough. Readers who find one of the notions obscure are of course free to think of one of the above principles as a definition of the less clear notion in terms of the clearer one.

I am myself sympathetic to the view that to be matter *just is* to be a material object that does not coincide with any of its proper parts, and, furthermore, that to be a material object *just is* to have parts (in the relevant sense of 'part'). I don't think such claims are in any interesting sense true by definition, but so long as they are true they would allow us to reduce all talk of matter and material objects to mereology.

[9] I am taking for granted that portions of matter are material objects, although this claim is not entirely uncontroversial. See Zimmerman (1995). But it is not controversial that portions of matter can be parts of other portions of matter, and of material objects more generally.

part with unit negative charge than with unit positive charge.[10]

An immediate worry for this proposal is that it seems to predict that all matter is microscopic, which it clearly isn't. But perhaps we can solve this problem with a dose of mereological essentialism. Say that x *has its* F *parts rigidly* just in case the parts of x that are F are such that, necessarily, x exists just in case all of them exist and, if x exists, then it has all of them as parts.[11] Say that x is *fundamental* just in case it has some property that figures in the laws of fundamental physics. We can now modify Hawthorne's proposal as follows: for a material object to be matter is for it to have its fundamental parts rigidly. This combination of essentialist and nomological criteria allows for macroscopic matter without assuming ANTI-SYMMETRY.

Unfortunately, the revised proposal faces two serious problems. The first is that it might turn out that no material object has any property that figures in the laws of fundamental physics, since it might turn out that fundamental physics is concerned with physical fields rather than with material objects. The proposal would then entail that there is no matter. But surely we know that there is matter (although maybe it will turn out not to be fundamental).

The second problem is that the proposal has unacceptable modal consequences. By being a proposal about *what it is* to be matter, it entails the corresponding necessitated biconditional. This reveals the view to be ambiguous: should we understand it *de dicto*, giving the description 'the laws of fundamental physics' narrow scope, or *de re*, giving the description wide scope? Either understanding is problematic. On the *de dicto* reading, the view has the problematic consequence that duplicates of actual matter would have failed to be matter in worlds where the fundamental laws concern properties other than those that actually figure in the fundamental laws. On the *de re* reading, the view has the problematic consequence that there could not have been alien matter: matter no part of which has any of the properties with which actual fundamental physics is concerned. So while Hawthorne may be right that not all material objects have properties that figure in physical laws (other than by mereological proxy), that insight does not afford us a plausible independent characterization of matter.

Let's take stock. Assuming ANTI-SYMMETRY, we can distinguish matter from other material objects in mereological terms. To the extent that no comparatively plausible and straightforward alternative account is forthcoming, we thereby have strong theoretical reasons to accept ANTI-SYMMETRY as a working hypothesis. I will do just that, since the principle also enjoys strong initial plausibility and we have seen no reason to question it. The theory that follows should demonstrate the fruitfulness of this decision.

[10] Ignore the fact that quarks have fractional charge.

[11] For present purposes, let x exists $=_{df}$ x is a material object. So understood, it should be uncontroversial that material objects exist contingently; see Williamson (1988).

3 The mereology of matter

Consider the following principle:

> UNIQUE FUSION: For any material objects xx, there is a unique material object y that *fuses* them.

where 'xx', 'yy', *etc.* are plural variables and

> x *fuses* $yy =_{df}$ every one of yy is part of x and every part of x overlaps one of yy.

UNIQUE FUSION is false, since c and s both fuse the parts of c. Yet the principle has played a central role in the literature on material objects, as classical mereology – arguably the mereological orthodoxy – is equivalent to UNIQUE FUSION given the transitivity of parthood.[12] What explains its appeal?

I suspect that the appeal of UNIQUE FUSION is explained by the truth of the following closely related principle:

> MATTER-RESTRICTED UNIQUE FUSION: For any material objects xx each of which is matter, there a unique material object y that is matter and fuses them.

In other words, classical mereology is true *when its quantifiers are restricted to matter.*[13] Since MATTER allows us to distinguish matter from other material objects in mereological terms, MATTER-RESTRICTED UNIQUE FUSION is equivalent to a claim solely about the mereology of material objects. And since, by MATERIALITY, every material object coincides with some matter, we can explain the appeal of classical mereology in terms of the widespread (though misguided) temptation to identify material objects that coincide.

It is notable that, among contemporary metaphysicians, a commitment to classical mereology is often accompanied by a commitment to some version of the vague idea that material objects are 'nothing more than' their parts.[14] Perhaps for some this comes from a sense that classical mereology makes the mereological structure of the world hopelessly boring; for others it perhaps comes from a thought that classical mereology requires an ontology too bloated to be believed without some deflationary gloss. Whatever its source, this perspective

[12] See Lewis (1991). Lewis's views in this area are difficult to characterize. On the one hand, he claims to accept classical mereology. On the other hand, the notion of part that he claims satisfies classical mereology is not the ordinary one, according to which things have different parts at different times, but rather a notion of 'atemporal parthood' in terms of which he claims the ordinary notion can be analyzed. Lewis is therefore best classified as holding a version of the ANTI-SYMMETRY-denying view discussed above (since ANTI-SYMMETRY comes out false for 'temporary parthood' on his analysis thereof), although he would also protest against my way of framing the debate.

A more 'three-dimensionalist' proponent of classical mereology might be Descartes, according to whom material objects are identical to (moving) regions of space and have all and only their subregions as their parts.

[13] Simons (1987) also discusses this idea.

[14] See Sider (2007).

sometimes leads metaphysicians to lose their grip on reality. They begin to suspect that talk of complex objects is just a roundabout way of talking about their (purported) atomic parts, and eventually conclude that we should reject such talk when doing serious metaphysics.[15] I have found in my own case that such mereological nihilism loses much of its allure once I conceive of the alternative not as classical mereology but instead as the acknowledgement of a rich and familiar hierarchy of objects built on a classical foundation of matter. So I encourage readers with nihilist sympathies to try such hierarchical conceptions on for size. It is to such conceptions that I will now turn.

4 Constitution

Your body is made of your limbs and organs, which are in turn made of your bones and tissues, all the way down to ultimate microscopic matter. This is the hierarchy of *building blocks*. You, your body, and the matter composing your body also form a hierarchy of coinciding objects. Call this the hierarchy of *immediate bases*. I take these hierarchies to be reasonably familiar, both from the sort of examples just given and from the literature on 'material constitution'. I propose to understand them mereologically:

> BUILDING BLOCK: A material object is a building block of another just in case it is an immediate part of it and does not coincide with it.
>
> IMMEDIATE BASIS: A material object is an immediate basis of another just in case it is an immediate part of it and coincides with it.

where

> x is an *immediate part of* $y =_{df}$ x is a proper part of y, x is not a proper part of any proper part of y, and y is not matter.[16]

Such straightforward mereological accounts of material constitution are hopeless if we reject ANTI-SYMMETRY and insist that coinciding material objects are parts of each other: if you, your body, and the matter composing your body were all coincident parts of each other, then neither you nor your body would have any immediate parts. Given the poor track record of alternative accounts of asymmetric relations of material constitution,[17] the availability of simple mereological accounts provides strong evidence for present framework.

The account is not without its surprises. We might naïvely have thought that (i) my heart is a building block of my body, (ii) my heart is part of my

[15] See Dorr (2005) and Sider (2013).

[16] The requirement that y not be matter is needed to capture the idea that bits of matter are unstructured mere aggregates of their parts. Without it, the result of 'subtracting' an atom from some matter would count as a building block of it, and any matter that fused infinitely many atoms would have an infinitely descending chain of building blocks, and so immediate parthood would fail to be well-founded. See also footnote 33.

[17] See Wasserman (2004).

torso, and (iii) my torso is part of my body. But these claims are inconsistent with BUILDING BLOCK, since my heart cannot be an immediate part of my body if my torso is mereologically in between. In response, one might reject (i) on the grounds that it is not my heart but rather my circulatory system that is a building block of my body. (Perhaps my heart is a building block of my circulatory system, and we are conflating a building block of a building block of my body for a building block of my body.) Or one might reject (ii): perhaps my torso is built not of organs but of smaller biological units, such as the bones and tissues between my waist and neck. (After all, some of my organs, such as my intestines, arguably overlap my torso without being parts of it, since not all of their parts overlap my torso.) Or one might reject (iii), on the grounds that once we distinguish the fact that my torso is a fusion of some but not all of my body's *building blocks* from the claim that my torso is a fusion of some but not all of my body's *matter*, the hypothesis that it is a part of my body becomes less gripping.

My response is to plead vagueness: our use of the word 'torso' in ordinary English is not disciplined enough to clearly resolve the question one way or another. But there is a general moral to be drawn – namely, that one of the following two principles is false:

> SUMS: Any two material objects have a fusion.
>
> LUBS: Any two material objects have a least upper bound.

where

> x is a *least upper bound* of y and $z =_{df}$ y and z are parts of x and x is part of anything that has both y and z as parts.

Let x and y be two of my building blocks – my heart and my liver, say – such that the fusion of the matter of x and the matter of y is a proper part of the matter of my body.[18] Since x and y are parts of my body, LUBS entails that they have a least upper bound z that is part of my body. Since x and y are building blocks of my body and parts of z, BUILDING BLOCK entails that z is identical to my body, by the definition of immediate parts. The definition of least upper bounds then entails that my body is part of anything that has both x and y as parts, and hence is part of any fusion of x and y. But my body is not part of any fusion of x and y, since my body has parts that overlap neither x nor y – namely, the parts of its matter that overlap neither the matter of x

[18] MATTER-RESTRICTED UNIQUE FUSION guarantees the existence of such a fusion. It also ensures that we can always speak of *the* matter of a material object. (Suppose for reductio that x coincides with distinct bits of matter y and z. Since coincidence is an equivalence relation, y and z coincide, and hence overlap the same things. *A fortiori*, they overlap the same matter. Classical mereology entails that material objects that overlap the same things are identical. So MATTER-RESTRICTED UNIQUE FUSION entails that bits of matter that overlap the same matter are identical, since, given the transitivity of parthood, it entails all theorems of classical mereology whose quantifiers have been restricted to matter. So y and z are identical, contradicting our supposition.)

nor the matter of y. So x and y have no fusion, contradicting SUMS. Which of SUMS and LUBS we give up will depend on whether we opt for a sparse or a plenitudinous material ontology. (The plenitudinous view explored in section 6 validates SUMS.)

Although I have spoken of the 'ultimate' matter of material objects, it does not follow from anything I have said so far that the material world bottoms out with matter. Since that principle seems to be part and parcel of the vision that is emerging, we may take it as a working hypothesis:

GROUND FLOOR: If x is matter, then every part of x is matter.

5 Persistence

We can apply the present mereological framework to debates about persistence. Assuming ANTI-SYMMETRY, the following principle allows us to give mereological expression to the idea that instantaneous 'temporal parts' are more fundamental than persisting material objects:[19]

PERDURANTISM: All persisting material objects perdure.

where

x *persists* $=_{df}$ x exists at more than one time.

x is *instantaneous* $=_{df}$ x exists at exactly one time.

x *perdures* $=_{df}$ x persists and at all times when it exists it has an instantaneous part with which it coincides.

Given our mereological characterization of matter, PERDURANTISM turns out to be equivalent to the thesis that all matter is instantaneous.[20] This is a welcome result, since the question of whether all matter is instantaneous is the sort of question we should be able to get some grip on by looking to contemporary physics. Now it is not entirely clear how to think about the persistence of matter according to our best microphysical theories – namely, relativistic quantum field theories – since, in spite of their staggering predictive success, their formalism has no agreed interpretation. Nevertheless, the trajectories observed in cloud chambers and in other experiments with particle accelerators strongly suggest

[19] Hawthorne (2006b) convincingly argues that extant characterizations of this idea are in various ways unsatisfactory. Fine (2006) has a very different strategy for saving the debate. Note that Sider (2001) does not accept PERDURANTISM as I have defined it (irrespective of whether we choose to understand 'existing at a time' in terms of his notion of location in space-time or in terms of his temporal counterpart theory).

[20] Left-to-right: if all persisting material objects perdure, then by MATERIALITY any non-instantaneous matter perdures, so any matter has an instantaneous part with which it coincides; but by MATTER no matter coincides with any of its *proper* parts; so any matter is itself instantaneous. Right-to-left: if all matter is instantaneous, then by MATERIALITY all material objects have an instantaneous object as a coincident part, and so perdure if they persist.

that matter is made of persisting (though not eternal) particles. For purposes of this paper, I will take this appearance of persisting matter at face value. Our current scientific evidence tells strongly against PERDURANTISM.[21]

Note that the falsity of PERDURANTISM fails to settle a number of interesting related questions. For example, it fails to settle the question of whether *any* material objects perdure, and, if so, which ones. It is doubtful that any objects recognized by common sense perdure. Assuming that there is no instantaneous matter, such objects would have to, at all times when they exist, have as a coincident part an instantaneous material object that is not matter. But what could such instantaneous objects be? Perhaps the continuity of the physical processes that underly the creation and destruction of ordinary artifacts suggest that there could be instantaneous tables and chair (although this is not obvious). But such instantaneous objects would be flukes, and it is hard to conceive of any persisting ordinary object having some such fleeting object as a coincident part at every moment of its career. Yet while common sense may recognize no perduring material objects, many metaphysicians recognize many more material objects than are recognized by common sense. Let us now turn to such views.

6 Plenitude

There is widely felt to be something objectionably unprincipled about believing in all and only the material objects recognized by common sense. Maybe this worry is ultimately spurious, in which case it poses no threat to the theory I have been advancing.[22] But for those, like me, who feel the force of the charge of arbitrariness, a crucial test of the theory of the material world I have been advocating is whether it can be fleshed out in some non-arbitrary way.

The most popular strategy for avoiding perceived arbitrariness is to adopt a plenitudinous ontology according to which material objects are exactly as plentiful as available 'modal profiles'. This idea is normally understood in something like the following way:

> FLAT PLENITUDE: Every eligible modal profile loosely corresponds to a unique possible material object.[23]

where

[21] An anonymous referee felt that this argument was too quick. After all, it is not as though perdurantists somehow failed to notice that particle physicists talk about electrons but not about instantaneous 'electron slices'. In reply: I admit to having no argument that particle physics is in the business of studying matter, but I doubt many would dispute it. At any rate, the novelty of the present argument lies not in that claim but in the claim that PERDURANTISM entails that all matter is instantaneous, which is a distinctive consequence of the mereology I have been developing. For example, it is not a consequence of the ANTI-SYMMETRY-denying mereology that perdurantists often presuppose.

[22] See Korman (2010) for a powerful reply to arguments from arbitrariness.

[23] Compare Hovda (2013).

> f is an *eligible modal profile* $=_{df}$ f is a function from world-instants to sets of possible material objects that exist at those world-instants, and f has a non-empty set in its range.[24]
>
> x *loosely corresponds to* $f =_{df}$ for all world-instants w, x fuses the members of $f(w)$ at w.

Unfortunately, FLAT PLENITUDE is inconsistent with the view I have been developing, since it is inconsistent with ANTI-SYMMETRY. Consider an eligible modal profile f that maps all but two world-instants w_1 and w_2 to the empty set. FLAT PLENITUDE entails that there is a unique possible material object x that loosely corresponds to this profile. It also entails that there is a possible material object x_1 that fuses $\{x\}$ at w_1 and exists at no other world-instants, a possible material object x_2 that fuses $\{x\}$ at w_2 and exists at no other world-instants, and a possible material object x^* that fuses $\{x_1\}$ at w_1, fuses $\{x_2\}$ at w_2, and exists at no other world-instants. By construction, x is part of x_1 at w_1 and x_1 is part of x^* at w_1.[25] By the transitivity of parthood and the relevant definitions, x^* also loosely corresponds to f, and so, by FLAT PLENITUDE, must be identical to x. So x and x_1 are a counterexample to ANTI-SYMMETRY at w_1.

The problem with FLAT PLENITUDE is that loose correspondence is defined in terms of an insufficiently discriminating notion of fusion. The informal idea of a fusion of a set of material objects is that of an object that has every member of the set as a part and has no *superfluous* parts. The definition of fusion above does not achieve this end, since it allows that I am a fusion of my matter in spite of the fact that my body is 'mereologically in between'. To make precise the prohibition on superfluous parts, we need to consider which objects must be part of a thing in order for it to fuse a set of material objects S. Every member of S must be part of it. The matter that fuses the matter of the members of S must also be part of it, since, by MATERIALITY, it must have as a part some matter with which it coincides. Finally, it must have all parts of the aforementioned objects as parts (by the transitivity of parthood), and it must have itself as a part (by the reflexivity of parthood). We can now define the relevant notion of fusion as follows:

> x *minimally fuses* $S =_{df}$ S is a set of material objects and, for all y, y is part of x just in case either y is part of a member of S, y is part of the material basis of S, or $y = x$.

where

> x is the *material basis of* $S =_{df}$ x is matter and x fuses $\{y$: y is matter and y is part of some member of $S\}$.

[24] I am using 'world-instant'-talk as an abbreviation for cumbersome quantification over world-time pairs and talk of what is true in a given world at a given time. There are subtle issues here; see Dorr and Goodman (unpublished).

[25] x fuses $S =_{df}$ x fuses the members of S.

I do not *minimally* fuse the set of my limbs and organs, since I have my body as a part, and my body is neither part of any of my limbs or organs, nor part of the material basis of the set of my limbs and organs, nor identical to me. Nor do I minimally fuse the singleton set of my matter, for the same reason. (It is plausible that my body minimally fuses both of these sets, but this claim does not follow from what I have said so far.)

The notion of minimal fusion allows us to formulate a more promising version of mereological plenitude:

> HIERARCHICAL PLENITUDE: Every eligible modal profile tightly corresponds to a unique possible material object.

where

> *x tightly corresponds to f* $=_{df}$ for all world-instants w, x minimally fuses the members of $f(w)$ at w.

Although no form of plenitude is forced upon us, HIERARCHICAL PLENITUDE is an attractive candidate for those of us who wish to avoid arbitrariness in our modal mereology. (Note that tight correspondence entails loose correspondence, since minimal fusion entails fusion, and hence HIERARCHICAL PLENITUDE entails the principle obtained from FLAT PLENITUDE by dropping the uniqueness requirement.)[26]

7 Consequences of hierarchical plenitude

A familiar problem for plenitudinous ontologies is that they often turn out to be inconsistent, Frege's theory of extensions being the most notorious example. So it is crucial to establish the consistency of HIERARCHICAL PLENITUDE. I will now describe a class of non-trivial models that validates the principle and also validates the preceding claims about the structure of the material world. The construction reveals that what might have seemed like a disparate collection of axioms in fact constitutes a unified picture of mereological reality.

Although the technical details are left to the appendix the basic idea of the construction is to model the hierarchical structure of the material world on the hierarchical structure of the pure sets. We begin with a set of world-instants and a function specifying which matter exists at which times in which worlds. We then, for every eligible modal profile to which no object already tightly corresponds, add such an object. We iterate this process into the transfinite and treat the resulting structure as a Kripke model for a quantified modal language.

[26] Note also that not every material object with immediate parts minimally fuses its immediate parts. Let $x_0, x_1, \ldots$ be an ω-sequence of coincident objects ordered by proper parthood, and let y be some matter not coincident with them. The minimal fusion of $\{x_0, x_1, \ldots, y\}$ will have its matter as its only immediate part but will not minimally fuse (the singleton set of) its matter.

The construction validates the reflexivity and transitivity of parthood. It also validates HIERARCHICAL PLENITUDE and ANTI-SYMMETRY, along with the following two modalized sufficient conditions for the identity of material objects:

> STRONG ANTI-SYMMETRY: If x is *possibly* part of y and y is *possibly* part of x, then $x = y$.
>
> INTENSIONALITY: If material objects x and y have proper parts and necessarily have the same proper parts, then $x = y$.

So while HIERARCHICAL PLENITUDE entails a great plenitude of material objects, it is also consistent with reasonable constraints on which objects there are.

We must now check that the construction is compatible with our mereological accounts of matter and material constitution. It can be verified that it validates MATTER, MATERIALITY, MATTER-RESTRICTED UNIQUE FUSION, GROUND FLOOR, SUMS and the following two attractive principles:

> FOUNDATION: Immediate parthood is well-founded.
>
> ESSENTIALITY: If x is part of y and y is matter, then necessarily, y exists only if it has x as a part.[27]

The validity of these last two principles further supports the present approach. The construction is also consistent with my claim that my body has my limbs and organs as non-matter building blocks and has its matter as its immediate basis, understood according to BUILDING BLOCK and IMMEDIATE BASIS. HIERARCHICAL PLENITUDE harmonizes nicely with our simple mereological accounts of matter and material constitution.

Here are some principles that come out false according to the construction. WEAK SUPPLEMENTATION immediately fails, as do the following two strengthenings of INTENSIONALITY:[28]

> EXTENSIONALITY: If material objects x and y have proper parts and have the same proper parts, then $x = y$.
>
> NECESSARY COINCIDENCE: If material objects x and y necessarily coincide, then $x = y$.

The failure of EXTENSIONALITY is not very surprising once one adopts a plenitudinous ontology. Suppose, as many philosophers believe, that for any material object and temporal interval throughout which it exists, there is a material object that we can think of as the 'restriction' of the original object to that interval. Let b_p be the restriction of my body to times at which it exists up to and including the present, and b_f be the restriction of my body to times at which it exists after and including the present. It is very natural to think

[27] Not all objects that have their parts essentially are matter: consider the object that now minimally fuses me but exists at no other world-instants.

[28] I am assuming that there are at least two world-instants at which there is matter.

that b_p and b_f have exactly the same proper parts – namely, all and only the parts of my body – and so are a counterexample to EXTENSIONALITY. Failures of NECESSARY COINCIDENCE are somewhat less straightforward. Fine (2000) offers a purported counterexample involving two letters written in different languages but composed of the same characters written on the same paper. More abstractly, if we suppose that for any matter there is an object that necessarily fuses the set of its temporal restrictions, then this object and the matter will be a counterexample to NECESSARY COINCIDENCE.[29] LUBS also comes out false in all non-trivial models, as expected.

8 Hylomorphism

The mereology I have been advocating has a number of similarities to theories of material objects inspired by the Aristotelian idea of *hylomorphism*, according to which a material object is a complex of 'matter' unified by a certain 'form'.[30] Both views emphasize the hierarchical structure of the material world. It is therefore worth comparing my view with what I take to be the best developed version of hylomorphism, namely Fine's (1999) *theory of embodiments*.

Fine's theory is in many respects similar to the one I have been developing. In particular

> [T]he majority of material objects, on our account, will submit to a hierarchical division into parts. Just as a car will have an engine, a chassis, and a body as immediate parts (these being the components of the rigid embodiment that is the current manifestation of the car), these immediate parts will themselves have further immediate parts, and so on all the way down until we reach the most basic forms of matter. Fine (1999, 72)

Fine also accepts a plenitudinous material ontology, the central principles of which are the following (which I have slightly simplified and somewhat modified to fit the present framework):

> RIGIDITY: For any material objects $x_1, x_2, \ldots$ related in some existence-entailing way R, there is a *rigid embodiment* $x_1, x_2, \ldots /R$ that, necessarily, exists just in case $x_1, x_2, \ldots$ are related by R, in which case it has all and only $x_1, x_2, \ldots$ as its building blocks.
>
> VARIABILITY: For any individual concept F, there is a *variable embodiment* $/F/$ that, necessarily, exists just in case something is F, in which case it has whatever is F as its immediate basis.

where

[29] Such objects will also perdure, assuming the matter in question persists.

[30] The Aristotelian notion of 'matter' corresponds not to matter (as I use the term) but rather to something like immediate parts. Perhaps Aristotle's notion of 'prime matter' corresponds to the notion of matter I have been operating with.

> F is an *individual concept* $=_{df}$ it is possible that some material object be F and it is impossible that more than one material object be F.

Fine offers a ham sandwich as a prototypical rigid embodiment, which we might take to be $b_1, h, b_2/S$, where b_1 and b_2 are each slices of bread, h is a slice of ham, and S is a three place relation of being arranged to form of a sandwich. He offers a river as a prototypical variable embodiment, which embodies some individual concept of portions of water. Most ordinary material objects of any complexity are taken to be variable embodiments of rigid embodiments. A car, for example, is taken to be a variable embodiment of an individual concept of rigid embodiments of car-parts variously related; these parts are in turn variable embodiments of rigid embodiments of their components. A plenitudinous modal mereology thereby emerges from the interplay of RIGIDITY and VARIABILITY. We take the part-whole relation to be the transitive, reflexive closure of the relation *either being a building block of or being the immediate basis of*.

As formulated above, Fine's view has two odd features. First, RIGIDITY entails that we can form rigid embodiments of the form $x, \ldots, (x, \ldots/R), \ldots/S$. Such embodiments have building blocks that are building blocks of other of their building blocks, since x is a building block of $x, \ldots/R$, which is in turn a building block of $x, \ldots, (x, \ldots/R), \ldots/S$. But this seems wrong, at least for the notion of an *immediate* building block, which Fine, like me, takes himself to be theorizing about. How can one thing be an immediate building block of another if another building block of the latter is 'compositionally in between'? Second, since only rigid embodiments have building blocks, it follows that, although the building blocks of a rigid embodiment can be contingent *parts* of a variable embodiment of which the rigid embodiment is the immediate basis, no material object has its *building blocks* as contingent parts. But surely we should allow for such variability. After all, aren't its engine, wheels, *etc.* the building blocks of my *car*, as opposed to (merely) building blocks of some unfamiliar rigid embodiment?

Fortunately, there is a natural way of modifying Fine's principles so as to avoid both of these unwanted results:

> RIGIDITY*: For any <u>pairwise modally disjoint</u> material objects $x_1, x_2, \ldots$ related in some existence-entailing way R, there is a *rigid embodiment* $x_1, x_2, \ldots/R$ that, necessarily, exists just in case $x_1, x_2, \ldots$ are related by R, in which case it has all and only $x_1, x_2, \ldots$ as its building blocks.

> VARIABILITY*: For any individual concept F, there is a *variable embodiment* $/F/$ that, necessarily, exists just in case something is F, in which case it <u>has as immediate parts</u> all and only the <u>immediate parts</u> of whatever is F.

where

> x and y are *modally disjoint* $=_{df}$ necessarily, neither x nor y is part of the other.

It turns out that VARIABILITY* and (with a minor caveat) RIGIDITY* are consequences of HIERARCHICAL PLENITUDE.[31] In this respect, then, Fine and I have much to agree on.

The major difference between our views concerns matter, of which Fine (1999) offers no account.[32] I will now argue that we cannot account for matter in terms of rigid embodiments. (I am assuming that matter has its parts essentially, and so does not involve variable embodiments.) The first problem is that Fine's theory fails to validate MATERIALITY. Consider the ham sandwich $b_1, h, b_2/S$. The sandwich is not matter. And no part of any of its building blocks b_1, b_2 and h coincides with it. Since Fine's view entails that the only parts of $b_1, h, b_2/S$ are itself and parts of its building blocks, it therefore entails that there is no matter with which it coincides, contradicting MATERIALITY. The second problem for understanding matter in terms of rigid embodiment is that, having replaced RIGIDITY with RIGIDITY*, we can prove that no rigid embodiment has the mereological behavior of infinitely divisible matter.[33]

In response, Fine might postulate a third primitive operation of 'aggregation' that allows us to form matter from other matter. We would then have to complicate the definition of parhood by taking aggregation into account. Perhaps everything I have said in terms of parthood could be consistently formulated in terms of this defined notion. But the resulting axioms would be unlovely. If the modal mereology I have proposed is indeed correct, then the comparative simplicity of formulating the view directly in terms of a dyadic relation of part-whole, rather than in terms of Finean operations of composition, seems

[31] We identify the rigid embodiment $x_1, x_2, \ldots /R$ with the object that tightly corresponds to the modal profile that maps every world w in which R holds among $x_1, x_2, \ldots$ to $\{x_1, x_2, \ldots\}$, and maps all other worlds to the empty set. We identify the the variable embodiment $/F/$ with the object that tightly corresponds to the modal profile that maps every world w to the set of immediate parts in w of the object (if there is one) that is F in w. Assuming BUILDING BLOCK, this identification scheme together with the definition of immediate parthood entails VARIABILITY* and, with one exception, RIGIDITY*. The exception concerns rigid embodiments x/E where the proposition that x is E is necessarily equivalent to the proposition that x exists. Both x and x/E tightly correspond to the modal profile that maps worlds in which x exists to $\{x\}$ and all other worlds to the empty set. So HIERARCHICAL PLENITUDE entails that they are identical. But RIGIDITY* entails that they are distinct, since nothing is a building block of itself and RIGIDITY* entails that x is a building block of x/E. This is not, however, a significant departure from Fine's view. We could easily either modify RIGIDITY* to rule out such trivial rigidifications, or modify HIERARCHICAL PLENITUDE in such a way that it generates them. (I should mention that Caplan et al. (2010) propose identifying $\{x\}$ with x/exists.)

[32] Fine has written about matter elsewhere, but is not straightforward to integrate what he says in other writings with his theory of embodiments.

[33] Consider some matter m that fuses infinitely many atoms. Every bit of matter that fuses all but one of the atoms is a proper part of m, but not a proper part of any proper part of m, and so must be one of m's building blocks in order to end up counting as part of m. The modal disjointness requirement then entails that the *only* building blocks of m are bits of matter with all but one of m's atoms. Iterating this line of reason, we end up with an infinite descent of rigid embodiments, at each stage pruning off one atom in every possible way, as it were. Taking the transitive, reflexive closure of the building block relation, the predicted parthood relation ends up being 'gunky' in the sense that none of the atoms ends counting as part of m. This gets m's mereological structure wrong, since we were assuming that it had atomic parts. (Things are even worse if m is in fact gunky, since the modal disjointness requirement then has the effect of saying that *nothing* can be a building block of m.)

to provide strong abductive grounds for preferring a purely mereological framework to a hylomorphic one. This is not to say that such considerations could never be outweighed. Perhaps hylomorphism can earn its keep in other ways, although I am skeptical.[34]

Another important difference between Fine's view and the plenitudinous view I have been exploring is that, according to the latter, the rigid embodiments x/F and x/G can be identical even if the conditions F and G are not identical, or even coextensive. For example, me/sitting = me/sitting-or-a-fried-egg. In general, $x_1, x_2, \ldots/R_1 = x_1, x_2, \ldots/R_2$ whenever $R_1 x_1 x_2 \ldots$ and $R_2 x_1 x_2 \ldots$ are necessarily equivalent. This sort of collapse prevents RIGIDITY* from falling afoul of Cantor's theorem. By contrast, Fine stipulates that $x_1, x_2, \ldots/R_1 = x_1, x_2, \ldots/R_2$ only if $R_1 = R_2$. It is unclear how his theory avoids generating a cardinality crash.

9 Further issues

One notable consequence of HIERARCHICAL PLENITUDE (together with ANTI-SYMMETRY and the assumption that there are at least two world-instants at which there exist material objects) is the falsity of the following principle:

> UNRESTRICTED FUSION: For any material objects xx, there is a material object that fuses them.

In particular, no material object fuses all material objects. (Suppose for reductio that x fuses all material objects. Let f be a modal profile to which x does not tightly correspond but such that $f(@) = \{x\}$, where @ is the actual world-instant; so long as there are two matter-containing world-instants, there will exist such an f. By HIERARCHICAL PLENITUDE, there is an object x^* that tightly corresponds to f. Since x does not tightly correspond to f, $x \neq x^*$. By the definition of tight correspondence, x^* minimally fuses $\{x\}$, and hence has x as a part. So by ANTI-SYMMETRY, x^* is not part of x, contradicting our assumption that x fuses all material objects.) We can make sense of this result by thinking of HIERARCHICAL PLENITUDE as encapsulating an iterative conception of what material objects there are, akin to the familiar iterative conception of sets. There fails to be a fusion of all material objects for the same reason that, according to the iterative conception of set, there fails to be a set of all sets: there is no stage at which such a fusion could be formed. The models described in the appendix give formal expression to this idea. Moreover, as in the set theoretic case, we do get the following restricted principle:

> SET-SIZED FUSION: For any material objects xx that can be put in a one-to-one correspondence with the members of some pure set, there is a material object that fuses them.

[34] Fine sometimes writes as if the part-whole relation is in need of some more fundamental metaphysical analysis, which he takes himself to be providing. In reply, it seems to me that the part-whole relation is much clearer than anything in terms of which it might be explained.

There is much more to be said about the connections between mereology, modality, and set theory that obtain if the view I have been outlining is correct, and I hope to explore these connections in future work. For now, I will simply list some topics for future investigation.

First, many philosophers of mathematics believe that the set theoretic universe is in certain respects indeterminate in structure, or perhaps 'indefinitely extensible', or in some other sense 'metaphysically unsettled'. It is much less common to hold analogous views about the concrete realm of material objects. Since HIERARCHICAL PLENITUDE entails that the structure of the set-theoretic universe is reflected in the realm of the concrete, it seems to destabilize such a split decision.

Second, many philosophers think that there is a certain amount of indeterminacy in the notion of metaphysical possibility. Supposing one recognizes a notion of entailment that applies to propositions, one might think that there are maximal consistent propositions that fail to be metaphysically possibly true. (A proposition is maximal consistent just in case it entails every proposition or its negation but never both.) For example, perhaps the proposition that I am a fried egg is entailed by some such proposition; i.e., perhaps it is metaphysically impossible without being inconsistent. The question then arises of which maximally specific propositions are metaphysically possibly true. HIERARCHICAL PLENITUDE offers a principled answer to this question: exactly those propositions (or equivalence classes thereof under the relation of mutual entailment) that satisfy the role for world-instants articulated by that principle. In this way, the notion of material object allows us to get purchase on the notion of metaphysical possibility. (Perhaps this is not surprising given the way that Kripke (1972) appeals to the modal properties of material objects in trying to explain the operative notion of possibility.)

Third, the mereology I have been describing is one according to which there are at least as many material objects as there are ordinals. This result is inconsistent with a cluster of commonly held set-theoretic assumptions – in particular, the standard axiomatization of impure set theory $ZFCU$ together with the assumption that there is a set of all material objects. There are a number of options here, but the issues subtle and beyond the scope of this paper.[35]

Fourth, HIERARCHICAL PLENITUDE (like FLAT PLENITUDE) seems to entail that there are more material objects than world-instants at which there exist material objects, since by Cantor's theorem there are more functions from matter-containing world-instants to sets of material objects that exist at those world-instants than there are matter-containing world-instants.[36] This consequence is in *prima facie* tension with various plausible principles of plenitude for metaphysical possibility. (Things get even trickier if we give up the assumption that the metaphysical possibilities form a set.)

[35] See Menzel (2014) for discussion.

[36] This argument is not air-tight. Menzel (2014) argues that we should allow for the existence of impure sets larger than any pure set, and that the power-set axiom does not apply to such 'wide' sets. The Cantorian argument then fails to go through, since the metaphysical possibilities may form a wide set to which the power-set axiom does to apply.

Finally, although here is not the place to go through the details, it turns out that the view I have been defending enables one to code up the impure set theory of material objects using only the resources of modal mereology with plural quantifiers. One might think that it thereby supports a reduction of set theory to mereology. Again, I leave discussion of such proposals for future work.[37]

10 Conclusion

I have shown how we can capture the familiar hierarchical structure of the material world, with matter at its foundation, solely in terms of the relation of part-whole as it applies to material objects. Together with modal notions, we can give a principled fleshing-out of the ontology of common sense. The resulting theory is both natural and parsimonious, and I think deserves serious consideration.

Appendix

Let W be a non-empty set and M be a complete Boolean algebra with operators $-$ and $\sqcup$. Let D_0 be a function from W to principal ideals of M with the zero-element removed, and let $\leq_0$ be a function from $w \in W$ to $\{\langle x, y\rangle \in D_0(w)^2 : y = \sqcup\{x, y\}\}$. Think of D_0 as specifying which matter exists at which world-instants, and $\leq_0$ as specifying the part-whole relation on matter. We now inductively extend the domain function and part-whole relation to include non-matter material objects.

<u>Definitions</u>

- S is *materially closed* $=_{df}$ $M \cap S \neq \emptyset$ and $\sqcup\{M \cap S\} \in S$
- S is *downward closed*$_\alpha$ *at* $w =_{df}$ $S = \{x : \text{for some } y \in S, \langle x, y\rangle \in \leq_\alpha(w)\}$.
- f is a *candidate*$_\alpha$ $=_{df}$ $\emptyset \subset dom(f) \subseteq W$ and, for all $w \in dom(f)$, $f(w)$ is materially closed and downward closed$_\alpha$ at w.
- f is the *profile*$_\alpha$ of $x =_{df}$ $dom(f) = \{w \in W : x \in D_\alpha(w)\}$ and, for all $w \in dom(f), f(w) = \{y \in D_\alpha(w) : \langle y, x\rangle \in \leq_\alpha(w)\}$.

[37] One immediate worry for such a reductive project is that in order to do physics we need to be able to talk not only about sets of material objects but also about sets of regions of space-time. We could answer this worry by adopting the view that regions of spacetime are matter, on which parthood coincides with the subregion relation.(One could go even further and accept analogous theses about configuration space or fiber-bundle spaces in order to assimilate quantum mechanics and gauge theories under such a reductive scheme; see Albert (1996) and Arntzenius (2012) for discussion.)

Such a reduction of set theory to mereology would not seem to answer Field's (1980) challenge to formulate physical theories without reference to individuals outside the ontology of those theories, since it would involve appealing to non-matter material objects where physics is concerned only with matter (insofar as it makes any reference to material objects).

- f is *eligible*$_\alpha =_{df}$ f is a candidate$_\alpha$ and not the profile$_\alpha$ of any $x \in \bigcup_w D_\alpha(w)$.

  Successor stages

- $D_{\alpha+1}(w) = D_\alpha(w) \cup \{f : f \text{ is eligible}_\alpha \text{ and } w \in dom(f)\}$.
- $\leq_{\alpha+1}(w) = \leq_\alpha(w) \cup \{\langle x, y\rangle : y \in D_{\alpha+1}(w) \backslash D_\alpha(w)$ and either $x = y$ or $x \in y(w)\}$.

  Limit stages

- Same as successor stages, replacing D_α and $\leq_\alpha$ with $\bigcup_{\alpha<\lambda} D_\alpha$ and $\bigcup_{\alpha<\lambda} \leq_\alpha$.

We then define a Kripke model $\langle W, @, D, \leq, M\rangle$ for a quantified modal language with a dyadic 'is part of' predicate and a monadic 'is matter' predicate, and whose quantifiers range over material objects. $D(w) = \bigcup_\alpha D_\alpha(w)$ and $\leq(w) = \bigcup_\alpha \leq_\alpha(w)$, which will normally be proper classes.

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