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Diamonds are Forever

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Abstract

We defend the thesis that every necessarily true proposition is always true. Since not every proposition that is always true is necessarily true, our thesis is at odds with theories of modality and time, such as those of Kit Fine and David Kaplan, which posit a fundamental symmetry between modal and tense operators. According to such theories, just as it is a contingent matter what is true at a given time, it is likewise a temporary matter what is true at a given possible world; so a proposition that is now true at all worlds, and thus necessarily true, may yet at some past or future time be false in the actual world, and thus not always true. We reconstruct and criticize several lines of argument in favor of this picture, and then argue against the picture on the grounds that it is inconsistent with certain sorts of contingency in the structure of time.

'Impossible is temporary' — Muhammed Ali

I

Nothing impossible has ever happened. If a proposition is necessarily true, then it is always true. Call this principle *Perpetuity*. Obvious as it sounds, *Perpetuity* has fallen on hard times. We aim to defend it.[1]

Three points of clarification. First: Some philosophers are *propositional eternalists*: they think that every true proposition is always true. On this view, *Perpetuity* is true but uninteresting, since it is uncontroversial that every necessarily true proposition is true. We think that even *propositional temporalists* — those who deny propositional eternalism — should accept *Perpetuity*. We will therefore proceed on the assumption that there are some temporarily true propositions.

Second: Some philosophers are nominalists and think that there aren't any propositions at all. This view also makes *Perpetuity* true but uninteresting. Yet surely nominalists should want to find some way of reframing the debate over *Perpetuity* that renders it non-trivial by their lights — one need not believe in propositions to wonder whether necessity entails eternity. One attractive reframing strategy involves replacing quantification over propositions with quantification into sentence position. The debate about *Perpetuity* will then be reframed as a debate about whether, for all p: if necessarily p, then always p.[2] We think this is also a perfectly intelligible debate. Since in what follows we will be freely intersubstituting φ and $\ulcorner$the proposition that φ is true$\urcorner$, it would be a trivial exercise to rewrite our discussion of *Perpetuity* using quantification into sentence position in place of quantification over propositions.[3]

Third: *Perpetuity* is a claim about *metaphysical* necessity. Given propositional temporalism, there are certainly other notions of necessity which do not entail eternity. For example, epistemic necessity does not entail eternity, since the proposition that there are dogs is known to be true but has not always been true. Similarly, the "historical" notion of necessity, on which truths about the past are automatically necessary, does not entail eternity, since there are non-eternal truths about the past (e.g., that it has rained at least once).

But how could something that was true yesterday be metaphysically impossible today, or something that is metaphysically necessary today be false tomorrow? We hope you share our judgment that it obviously couldn't be. If you do, then you might be surprised (as we were) to learn that this judgment of obviousness is far from universal. In fact, *Perpetuity* is inconsistent with most well-developed theories of the interaction of tense and modal operators. Admittedly, many of those who develop combined modal-temporal logics do not specify that they are talking about metaphysical necessity, and indeed some (e.g. Thomason 1984) are explicitly concerned with historical necessity, which seems on its face to be something altogether different from metaphysical necessity.[4] But two of the most prominent treatments of the interaction of tense and modality — David Kaplan's logic of demonstratives (1989), which is the starting point for much subsequent theorizing about tense and modality in the philosophy of language and formal semantics, and Kit Fine's modal tense logic (1977), which has been influential in the metaphysics literature — are explicitly concerned with metaphysical necessity.[5] While neither of these authors explicitly considers *Perpetuity*, as we shall see they are committed to its falsity. Meanwhile, many treatments of tense and modality in the linguistics literature are intended to apply to all available readings of modal adverbs in natural language, and seem to entail that *Perpetuity* is false no matter how we resolve the context-sensitivity of 'necessarily'.[6,7]

One underlying motivation behind these views is the desire for a symmetric treatment of modality and tense. For consider the converse of *Perpetuity*: the claim that every proposition that is always true is necessarily true. This claim is clearly false, since there are metaphysically contingent propositions that are always true, such as the proposition that sometimes it rains.[8] Thus *Perpetuity*, if true, constitutes an important asymmetry between necessity and eternity.

Since all of the aforementioned authors articulate their views in model-theoretic terms, getting a more precise understanding of their view will require a model-theoretic detour. In §II we define a class of "product models" for a simple formal language with modal and tense operators, and show that *Perpetuity* is inconsistent with propositional temporalism in these models. In §III we present a more general class of "relational structures", the logic of which we will treat as uncontroversial in what follows. This will allow us to isolate a principle, *Symmetry,* which explicitly captures the aspect of our opponents' view responsible for the incompatibility of *Perpetuity* with propositional temporalism. In §IV–§VII we consider and respond to four arguments for *Symmetry*: although these arguments have not actually been given in the literature, they are valid on the class of relational structures, and their premises are both philosophically interesting and true in all product models. (We will not appeal to model-theoretic considerations in assessing these arguments, so those who would prefer not to bother with model theory can skip §§II–III, referring to §III only for the statements of *Symmetry* and an equivalent principle, *Supervenience.*) In §§VIII–IX we argue against *Symmetry* for reasons having to do with contingency in what times there are. In §X we conclude by considering and rejecting the suggestion that the whole debate is merely verbal.

II

In this section we will define a class of "product models" for a formal language $\mathcal{L}$ containing the Boolean connectives $\neg$ and $\wedge$, the modal operator $\Box$ ('necessarily'), the tense operator A ('always'), and infinitely many propositional variables p_i and corresponding universal quantifiers $\forall p_i$. We will adopt standard abbreviations for $\vee$, $\rightarrow$, $\leftrightarrow$, $\exists$, and $\Diamond$; Sφ ('sometimes φ') abbreviates $\neg A \neg \varphi$.[9] We can formalize *Perpetuity* in $\mathcal{L}$ as $\forall p(\Box p \rightarrow Ap)$, assuming a scheme for translating $\mathcal{L}$ into English on which $\forall p_i$ is translated as 'For every proposition p_i', while occurrences of p_i in sentence position are translated as 'p_i is true'.[10]

A *product frame* is any ordered pair $\langle W \times T, \langle \alpha, \eta \rangle \rangle$ such that $\langle \alpha, \eta \rangle \in W \times T$. An *assignment function* on a product frame is a function from propositional variables to subsets of $W \times T$. A *product model* is a pair $\langle \langle W \times T, \langle \alpha, \eta \rangle \rangle, [\![\cdot]\!] \rangle$, where $\langle W \times T, \langle \alpha, \eta \rangle \rangle$ is a product frame and $[\![\cdot]\!]$ is the unique "interpretation function" that, given any assignment function g, maps each $\mathcal{L}$-formula φ to a subset $[\![\varphi]\!]^g$ of $W \times T$ in accordance with the following recursive definition:

$$\begin{aligned}
[\![p_i]\!]^g &= g(p_i) \\
[\![\neg\varphi]\!]^g &= (W \times T) - [\![\varphi]\!]^g \\
[\![\varphi \wedge \psi]\!]^g &= [\![\varphi]\!]^g \cap \psi]\!]^g \\
[\![\Box\varphi]\!]^g &= \{\langle w, t\rangle : \langle w', t\rangle \in [\![\varphi]\!]^g \text{ for all } w' \in W\} \\
[\![A\varphi]\!]^g &= \{\langle w, t\rangle : \langle w, t'\rangle \in [\![\varphi]\!]^g \text{ for all } t' \in T\} \\
[\![\forall p_i \varphi]\!]^g &= \{\langle w, t\rangle : \langle w, t\rangle \in [\![\varphi]\!]^h \text{ for all assignment functions } h \text{ that agree} \\
&\qquad \text{with } g \text{ on all variables distinct from } p_i\}.
\end{aligned}$$

φ is *true in* the product model $\langle \langle W \times T, \langle \alpha, \eta \rangle \rangle, [\![\cdot]\!] \rangle$ on an assignment g just in case $\langle \alpha, \eta \rangle \in [\![\varphi]\!]^g$. A closed sentence is true in a product model just in case it is true on some (or equivalently, on every) assignment. The *product logic* is the set of sentences true in all product models.

Perpetuity is true in a product model $\langle \langle W \times T, \langle \alpha, \eta \rangle \rangle, [\![\cdot]\!] \rangle$ just in case $T = \{\eta\}$, since when $W \times \{\eta\}$ is assigned to p, $\Box p$ is true, while Ap is false if T has more than one member. This is also the condition for $\forall p(p \to \mathrm{A}p)$ ('Every true proposition is always true') to be true in a product model. So, as advertised, *Perpetuity* turns out to be equivalent to propositional eternalism in the product logic.[11]

There are some additions to $\mathcal{L}$ that look natural from the perspective of product models, although (as we shall see) their interpretation in other kinds of models we discuss below raises difficult philosophical issues. Most straightforwardly, we can add operators @ ('actually') and N ('now'), with the following semantic clauses:

$$[\![@\varphi]\!]^g = \{\langle w, t\rangle : \langle \alpha, t\rangle \in [\![\varphi]\!]^g\}$$
$$[\![\mathrm{N}\varphi]\!]^g = \{\langle w, t\rangle : \langle w, \eta\rangle \in [\![\varphi]\!]^g\}$$

In §IV and §V we consider arguments against Perpetuity formulated using these operators. We could also enrich $\mathcal{L}$ with devices for explicitly talking about worlds and times, as follows. We add countably many "world variables" w_i, and "time variables" t_i have assignment functions assign a member of W to each world variable and a member of T to each time variable and have $\forall w_i$ and $\forall t_i$ vary these assignments in the obvious way. We can then add new predicates 'Actualized' and 'Present' that take, respectively, a world variable or a time variable as arguments, interpreted as follows:

$$[\![\mathrm{Actualized}(w_i)]\!]^g = \{\langle g(w_i), t\rangle : t \in \mathrm{T}\}$$
$$[\![\mathrm{Present}(t_i)]\!]^g = \{\langle w, g(t_i)\rangle : w \in \mathrm{W}\}$$

Instead, or in addition, we could add an 'At' operator that takes a world or time variable and a formula as arguments, interpreted as follows:

$$[\![\mathrm{At}\ w_i\ \varphi]\!]^g = \{\langle w,\ t\rangle : \langle g(w_i), t\rangle \in [\![\varphi]\!]^g\}$$
$$[\![\mathrm{At}\ t_i\ \varphi]\!]^g = \{\langle w,\ t\rangle : \langle w, g(t_i)\rangle \in [\![\varphi]\!]^g\}$$

If we have both 'Present' and 'At' in the language, the above clauses make 'Present(t_i)' logically equivalent to '$\forall p(p \leftrightarrow \mathrm{At}\ t_i\ p)$', and also make 'At t_i φ' logically equivalent to both 'A(Present(t_i) $\to \varphi$)' and 'S(Present(t_i) $\wedge\ \varphi$)'; similarly for 'Actualized'. But be warned that such equivalences are not uncontroversial, since taken together they impose severe constraints on the extent to which there can be contingency in the composition of the time series, as we discuss in §IX.[12]

Adding world- and time-quantifiers makes @ and N redundant in a certain sense: every sentence φ containing N is logically equivalent to $\exists t(\mathrm{Present}(t) \wedge \varphi^*)$, where t is a time-variable that doesn't occur in φ and φ^* is the result of substituting 'At t' for every occurrence of N in φ. Similarly for @ and 'At w'.

Moreover, as Fine (1977) shows, the addition of quantification over times and worlds is also expressively redundant in the product logic. In place of such quantification, we can quantify over 'time-propositions' — intuitively, propositions equivalent to a particular time's being present — and 'world-propositions' —

intuitively, propositions equivalent to particular world's being actualized — where these notions are defined as follows:

$$\text{World Prop}(\varphi) =_{\text{df}} \Diamond \mathrm{A}(\varphi \wedge \forall p(p \rightarrow \Box(\varphi \rightarrow p)))$$
$$\text{Time Prop}(\varphi) =_{\text{df}} \mathrm{S}\Box(\varphi \wedge \forall p(p \rightarrow \mathrm{A}(\varphi \rightarrow p)))$$

The upshot of these definitions is that $[\![\text{WorldProp}(\varphi)]\!]^g = \mathrm{W} \times \mathrm{T}$ if, for some $w \in \mathrm{W}$, $[\![\varphi]\!]^g = \{\langle w, t\rangle : t \in \mathrm{T}\}$, and $= \varnothing$ otherwise; likewise, $[\![\text{TimeProp}(\varphi)]\!]^g = \mathrm{W} \times \mathrm{T}$ if, for some $t \in \mathrm{T}$, $[\![\varphi]\!]^g = \{\langle w, t\rangle : w \in \mathrm{W}\}$, and $= \varnothing$ otherwise. As Fine (1977: 167) puts it, 'the instant-propositions and the world-propositions are the temporal and modal cross-sections, respectively, of the two-dimensional instant-world manifold'. To eliminate world and time quantifiers from a formula without free world or time variables, we proceed as follows: First, relabel the variables so that no index is shared between variables of different types. Second, eliminate 'At' in favor of 'Present' and 'Actualized' as described above. Third, replace each subformula of the form $\forall w_i(\ldots)$ with $\forall p_i(\text{WorldProp}(p_i) \rightarrow \ldots)$, each subformula of the form $\forall t_i(\ldots)$ with $\forall p_i(\text{TimeProp}(p_i) \rightarrow \ldots)$, and each subformula of the form $\text{Present}(t_i)$ or $\text{Actualized}(w_i)$ with p_i. It is straightforward to verify that the resulting formula has the same semantic value as the original formula on every assignment in every model. But be forewarned that these equivalences do not hold in any of the other classes of models we will be considering below.

The fact the negation of *Perpetuity* follows from propositional temporalism in the product logic does not by itself constitute anything like an *argument* against *Perpetuity* that could be set against its seemingly obvious truth. Perhaps model-theoretic elegance is some sort of guide to truth; but there are comparably elegant classes of models which validate *Perpetuity* without validating propositional eternalism. For example, Montague (1973) uses what we will call 'Montagovian models' — models also based on product frames, but with a clause for $\Box$ equivalent in our notation to:

$$[\![\Box\varphi]\!]^g = \{\langle w, t\rangle : \langle w', t'\rangle \in [\![\varphi]\!]^g \text{ for all } w' \in \mathrm{W} \text{ and } t' \in \mathrm{T}\}.$$

That is, $[\![\Box\varphi]\!]^g = \mathrm{W} \times \mathrm{T}$ if $[\![\varphi]\!]^g = \mathrm{W} \times \mathrm{T}$ and $= \phi$ otherwise; so clearly, $\mathrm{A}p$ is true on an assignment whenever $\Box p$ is. In a footnote to Montague's clause, his editor Richmond Thomason writes 'Here, $\Box$ is interpreted in the sense of "necessarily always"' (Montague 1974: 259). We disagree: Montague explicitly states that 'necessarily' is to be translated as '$\Box$', and the natural explanation for his interpreting $\Box$ in the above way is a desire to respect *Perpetuity*.

The additions to $\mathcal{L}$ discussed above can be interpreted in Montagovian models in the same way as in product models. These interpretations are completely natural for the temporal additions (N, $\forall t$, 'Present', and 'At t'). They are less natural for the modal additions (@, $\forall w$, 'Actualized' and 'At w'), since for example we will lose the equivalence of $\Box p$ and $\forall w$ At w p. (In §V and §VI we will discuss some other ways of interpreting possible-worlds talk.) Note too that Fine's reduction of the expanded language to $\mathcal{L}$ behaves pathologically in Montagovian models:

$\exists p(\mathrm{TimeProp}(p))$ and $\exists p(\mathrm{WorldProp}(p))$ are both false in any Montagovian model where T has more than one member. More generally, when N, or $\forall t$ and 'Present', or $\forall t$ and 'At t', or @, or $\forall w$ and 'At w' are interpreted in Montagovian models in the ways described above, some formulae without free time or world variables are not equivalent to any $\mathcal{L}$-formula.[13] This suggests a broader lesson: for proponents of *Perpetuity*, quantification over times provides a kind of co-ordination between different possible world-histories that cannot be expressed using only standard modal and temporal operators. It is open to them to think that sentences that ineliminably involve these additional resources are problematic in a way that $\mathcal{L}$-sentences are not; for example, perhaps time-quantification gives rise to vagueness or indeterminacy, in a way that temporal operators do not, when quantifying into the scope of modal operators.

III

In this section we will define a more general class of models whose logic we will treat as uncontroversial in what follows. Exploring the connection between these models and product models leads us to a principle — *Symmetry* — which is valid in the product logic and whose conjunction with propositional temporalism uncontroversially implies the falsity of *Perpetuity*. Isolating this principle thereby allows us to prescind from model theoretic considerations in the rest of the paper, considering instead the comparative merits of *Symmetry* and *Perpetuity*.

A *relational structure* is a quadruple $\mathcal{S} = \langle \mathrm{I}, \approx_\Box, \approx_\mathrm{A}, \iota\rangle$, where $\approx_\Box$ (the 'modal accessibility relation') and $\approx_\mathrm{A}$ (the 'temporal accessibility relation') are each equivalence relations on I, and ι (the 'home point') is a member of I. Given a relational structure $\langle \mathrm{I}, \approx_\Box, \approx_\mathrm{A}, \iota\rangle$ and an assignment function g mapping propositional variables to subsets of I, we define the interpretation function $[\![\cdot]\!]^g$ from formulae of $\mathcal{L}$ to subsets of I as follows:

$$\begin{aligned}
[\![p_i]\!]^g &= g(p_i)\\
[\![\neg\varphi]\!]^g &= \mathrm{I} - [\![\varphi]\!]^g\\
[\![\varphi \wedge \psi]\!]^g &= [\![\varphi]\!]^g \cap [\![\psi]\!]^g\\
[\![\Box\varphi]\!]^g &= \{x \in \mathrm{I} : y \in [\![\varphi]\!]^g \text{ for all } y \text{ such that } x \approx_\Box y\}\\
[\![\mathrm{A}\varphi]\!]^g &= \{x \in \mathrm{I} : y \in [\![\varphi]\!]^g \text{ for all y such that } x \approx_\mathrm{A} y\}\\
[\![\forall p_i\varphi]\!]^g &= \{j \in \mathrm{I} : j \in [\![\varphi]\!]^h \text{ for all } h \text{ that agree with } g \text{ on variables other than } p_i\}
\end{aligned}$$

φ is true on an assignment g in $\langle \mathrm{I}, \approx_\Box, \approx_\mathrm{A}, \iota\rangle$ if and only if $\iota \in [\![\varphi]\!]^g$; a closed sentence is true in a relational structure if and only if it is true on some assignment; the *background logic* is the set of all sentences true in every relational structure.

We will now consider some important subclasses of relational structures.

(i) Let a *connected* relational structure be one in which every point in I can be reached from ι by some finite sequence of points, each bearing $\approx_\Box$ or $\approx_\mathrm{A}$ to its predecessor. Clearly the same sentences are true in a relational structure as are true in the connected relational structure derived from it by throwing away all the points not reachable from ι by such a finite sequence; thus the background logic is also the logic of the class of connected relational structures.

(ii) Let a *product structure* be a relational structure of the form $\langle W\times T, =_2, =_1, \langle \alpha, \eta\rangle\rangle$, where $\alpha \in W$, $\eta \in T$, $\langle w, t\rangle =_1 \langle w', t'\rangle$ if and only if $w = w'$, and $\langle w, t\rangle =_2 \langle w', t'\rangle$ if and only if $t = t'$.[14] A product structure has the same interpretation function as the product model based on $\langle W\times T, \langle \alpha, \eta\rangle\rangle$. So the background logic is contained in the product logic.

(iii) Let a *Montagovian structure* be a relational structure of the form $\langle W\times T, (W\times T) \times (W\times T), =_1, \langle \alpha, \eta\rangle\rangle$—i.e. the same as a product structure but with a universal modal accessibility relation. A Montagovian structure has the same interpretation function as the Montagovian model based on the product frame $\langle W\times T, \langle \alpha, \eta\rangle\rangle$. So the background logic is contained in the logic of Montagovian models.

(iv) Let a *generalized product structure* be a relational structure of the form $\langle D, =_2, =_1, \langle \alpha, \eta\rangle\rangle$, where D is some set of ordered pairs and $\langle \alpha, \eta\rangle \in D$.[15]

(v) Let a *generalized Montagovian* structure be one of the form $\langle D, D \times D, =_1, \langle \alpha, \eta\rangle\rangle$, where D is some set of ordered pairs and $\langle \alpha, \eta\rangle \in D$.

Every relational structure with a universal modal accessibility relation is isomorphic to a generalized Montagovian structure. (The function mapping each point x to the ordered pair $\langle [x]_{\approx_A}, x\rangle$, where $[x]_{\approx_A}$ is the $\approx_A$-equivalence class containing x, is an isomorphism between the original structure and the generalized Montagovian structure whose set of points is the image of that function.) And every relational structure with a universal modal accessibility relation in which all the equivalence classes under $\approx_A$ are equinumerous is isomorphic to a Montagovian structure. (Choose an arbitrary equivalence relation $\approx_?$ such that, for all x and y there is a unique y' in $[y]_{\approx_A}$ such that $x \approx_? y'$, and then consider the function mapping each point x to the ordered pair $\langle [x]_{\approx_A}, [x]_{\approx_?}\rangle$.)

We can similarly give intrinsic characterizations, up to isomorphism, of generalized product structures and product structures. Say that a point x in a relational structure is *unaccompanied* if x is the only point both modally and temporally accessible from x. Clearly, every point in a generalized product structure is unaccompanied. Moreover, every relational structure in which every point is unaccompanied is isomorphic to a generalized product structure. Given any relational structure $\mathcal{S} = \langle I, \approx_\Box, \approx_A, \iota\rangle$, let $W_\mathcal{S} = I/\approx_\Box$ (the set of equivalence classes of I under $\approx_\Box$), $T_\mathcal{S} = I/\approx_A$, $\alpha_\mathcal{S} = [\iota]_{\approx_\Box}$ (the member of $I/\approx_\Box$ containing ι), and $\eta_\mathcal{S} = [\iota]_{\approx_A}$. Define $f_\mathcal{S} : I \to W_\mathcal{S} \times T_\mathcal{S}$ by $f_\mathcal{S}(j) = \langle [j]_{\approx_\Box}, [j]_{\approx_A}\rangle$, and let $D_\mathcal{S} = \{f_\mathcal{S}(j) : j \in I\}$. If every point in $\mathcal{S}$ is unaccompanied, $f_\mathcal{S}$ is one-to-one. Moreover $x \approx_\Box y$ if and only if $f_\mathcal{S}(x) =_2 f_\mathcal{S}(y)$, $x \approx_A y$ if and only if $f_\mathcal{S}(x) =_1 f_\mathcal{S}(y)$, and $f_\mathcal{S}(\iota) = \langle \alpha_\mathcal{S}, \eta_\mathcal{S}\rangle$, so $f_\mathcal{S}$ is an isomorphism from $\mathcal{S}$ to the generalized product structure $\langle D_\mathcal{S}, =_2, =_1, \langle \alpha_\mathcal{S}, \eta_\mathcal{S}\rangle\rangle$.

Turning to product structures: say that a point x in a relational structure is *square-completing* just in case whenever $x \approx_\Box y_1$ and $x \approx_A y_2$, there is some z such that $y_1 \approx_A z$ and $y_2 \approx_\Box z$. Every point in a product structure is square-completing (as well as unaccompanied). Moreover, every connected relational structure in which every point is both unaccompanied and square-completing is isomorphic to a product structure. This follows from the fact that whenever x and y are two points in a connected relational structure $\mathcal{S}$ where every point is square-completing, there is a point modally accessible from y and temporally accessible from x.[16] Hence every member of $\mathrm{W}_{\mathcal{S}}$ overlaps every member of $\mathrm{T}_{\mathcal{S}}$, so that $\mathrm{D}_{\mathcal{S}}$ (the image of the function $f_{\mathcal{S}}$ defined above) is the full Cartesian product $\mathrm{W}_{\mathcal{S}} \times \mathrm{T}_{\mathcal{S}}$.

The two properties of points we have just isolated correspond to the following pair of sentences of $\mathcal{L}$:

Symmetry: Every falsehood necessitates something that is never true when it is.

$$\forall p(\neg p \to \exists q(\Box(p \to q) \wedge \mathrm{A}(p \to \neg q)))$$

Church-Rosser: Whatever could always be true always could be true.

$$\forall p(\Diamond \mathrm{A} p \to \mathrm{A} \Diamond p)$$

In any relational structure, the unaccompanied points are all and only those where *Symmetry* is true, and the square-completing points are all and only those where *Church-Rosser* are true.[17]

(*Proofs*: (i) Suppose *Symmetry* is true at x, $y \approx_\Box x$, and $y \approx_\mathrm{A} x$. Let $g(p) = \{y\}$. Then $x \notin [\![\exists q(\Box(p \to q) \wedge \mathrm{A}(p \to \neg q))]\!]^g$, since for any q-variant g' of g, $x \notin [\![\Box(p \to q)]\!]^{g'}$ if $y \notin g'(q)$ while $x \notin [\![\mathrm{A}(p \to \neg q)]\!]^{g'}$ if $y \in g'(q)$. Since *Symmetry* is true at x, $x \notin [\![\neg p]\!]^g$, so $x \in g(p)$, so $x = y$. Hence x is unaccompanied. (ii) Suppose x is unaccompanied and $x \notin g(p)$. Let g' be the q-variant of g that assigns to q the set of all points that are either modally accessible from x and in $g(p)$, or temporally accessible from x and not in $g(p)$. Clearly $x \in [\![\Box(p \to q)]\!]^{g'}$. Also $x \in [\![\mathrm{A}(p \to \neg q)]\!]^{g'}$, since the only points in both $g'(p)$ and $g'(q)$ are modally accessible, but also distinct, from x, and hence not temporally accessible from x. So $x \in [\![\exists q(\Box(p \to q) \wedge \mathrm{A}(p \to \neg q)]\!]^g$, and thus *Symmetry* is true at x. (iii) Suppose *Church-Rosser* is true at x, $y_1 \approx_\Box x$, and $y_2 \approx_\mathrm{A} x$. Let $g(p) = [y_1]_{\approx_\mathrm{A}}$. Since $y_1 \in [\![\mathrm{A}p]\!]^g$, $x \in [\![\Diamond \mathrm{A}p]\!]^g$, so by *Church-Rosser*, $x \in [\![\mathrm{A}\Diamond p]\!]^g$, and thus $y_2 \in [\![\Diamond p]\!]^g$. Hence there is some z modally accessible from y_2 that belongs to $g(p)$, and so is temporally accessible from y_1: thus x is square-completing. (iv) Suppose x is square-completing and $x \in [\![\Diamond \mathrm{A}p]\!]^g$. Then there is a point y_1 modally accessible from x such that every point temporally accessible from y_1 is in $g(p)$. Consider any y_2 temporally accessible from x. Since x is square-completing, there is a point z that is both temporally accessible from y_1 (so $z \in g(p)$) and modally accessible from y_2 (so $y_2 \in [\![\Diamond p]\!]^g$). Since this holds for all such y_2, $x \in [\![\mathrm{A}\Diamond p]\!]^g$; hence *Church-Rosser* is true at x.)

It follows immediately from this result that every connected relational structure in which all the sentences that result from prefixing any sequence of $\Box$s and As to *Symmetry* or *Church-Rosser* are true is one in which every point is unaccompanied and square-completing, and hence isomorphic to a product structure. As it turns out, three of these sentences, namely $\Box$A(*Symmetry*), $\Box$(*Church-Rosser*), and A(*Church-Rosser*), suffice in the background logic to imply all the rest — and thus, to imply the entire product logic.[18,19]

Perpetuity is true in a relational structure just in case every point temporally accessible from ι is also modally accessible from ι. Propositional eternalism, meanwhile, is true just in case no point other than ι is temporally accessible from ι. These conditions will coincide in any relational structure whose home point is unaccompanied, i.e. in any relational structure in which *Symmetry* is true. This fact makes *Symmetry* a natural principle to focus on, since it seems to at least partially articulate the idea that time and modality interact in a symmetric way, it is valid in the product logic, and in the background logic it implies the collapse of *Perpetuity* to propositional eternalism.

In fact very little of the background logic is needed to establish this implication, as we can see from the following direct argument. Suppose *Symmetry* and *Perpetuity* are both true. Let p be any falsehood. By *Perpetuity*, $\forall q(\Box(p \to q) \to \mathrm{A}(p \to q))$, while by *Symmetry*, $\exists q(\Box(p \to q) \wedge \mathrm{A}(p \to \neg q))$. Combining these two formulae using standard quantifier reasoning, we have $\exists q(\mathrm{A}(p \to q) \wedge \mathrm{A}(p \to \neg q))$. Given the agglomeration of A over conjunction, this implies $\exists q\mathrm{A}((p \to q) \wedge (p \to \neg q))$, which is equivalent to $\exists q\mathrm{A}\neg p$ by substitution of tautological equivalents. But $\exists q\mathrm{A}\neg p$ obviously entails $\mathrm{A}\neg p$. Generalizing, we have $\forall p(\neg p \to \mathrm{A}\neg p)$, which is equivalent to propositional eternalism.

Symmetry is not the easiest principle to think about. Fortunately, it is equivalent in the background logic to the following structurally more familiar principle:

> *Supervenience*: Every truth is necessitated by a permanent truth.
>
> $$\forall p(p \to \exists q(\Box(q \to p) \wedge \mathrm{A}q))$$

(*Proof*: $\exists q(\Box(q \to p) \wedge \mathrm{A}q)$ is logically equivalent to $\exists q(\Box(q \to p) \wedge \mathrm{A}(\neg q \to p))$, since any q that witnesses the former witnesses the latter, while if q witnesses the latter, $p \vee q$ witnesses the former. *Supervenience* is therefore equivalent to $\forall p(p \to \exists q(\Box(q \to p) \wedge \mathrm{A}(\neg q \to p)))$; substituting $\neg p$ for p and $\neg q$ for q and contraposing the conditionals yields *Symmetry*.)

Supervenience doesn't wear its symmetry on its sleeve in the way that *Symmetry* does. Nevertheless, since *Supervenience* is equivalent to something equivalent to its mirror image (namely *Symmetry*), and the mirror image of anything valid in the background logic is also valid in the background logic, *Supervenience* is equivalent in the background logic to its mirror image:

> *Supervenience**: For every truth, there is a necessary truth that is never true without it being true.
>
> $$\forall p(p \to \exists q(\mathrm{A}(q \to p) \wedge \Box q))$$

This equivalence helps to bring out how *Symmetry* goes beyond the mere denial of the combination of *Perpetuity* with propositional temporalism: not only are there are temporary necessary truths, there are necessary truths of arbitrarily short temporal extent.

But why would anyone believe these principles? In the next four sections, we will consider some possible arguments.

IV

Perhaps the most straightforward argument for *Symmetry* turns on the following principle:

NOW Every proposition is, necessarily, true just in case it is now true.
$\forall p \Box (p \leftrightarrow \mathrm{N}p)$

Supervenience, and hence *Symmetry*, follows from NOW together with the following relatively uncontroversial additional premise:

$\mathrm{RIG_N}$ Everything true now is always true now.
$\forall p (\mathrm{N}p \rightarrow \mathrm{AN}p)$

For suppose that p is true. Then by NOW, the proposition that p is now true is true; by $\mathrm{RIG_N}$ it is a permanent truth, and by NOW it necessitates p. Thus every truth is necessitated by a permanent truth.

Given NOW, $\mathrm{RIG_N}$, and propositional temporalism, we can generate specific counterexamples to *Perpetuity*. Suppose p is temporarily true. Then the proposition that p is true if and only if p is now true is also temporarily true, since it is true exactly when p is true. But by NOW, this biconditional is necessarily true, and is therefore a counterexample to *Perpetuity*.[20]

Those who accept NOW will not think that the proposition it expresses is *always* true. Rather, the interesting, non-arbitrary view in the vicinity is that the modal status which NOW attributes to the present time is one which, necessarily, every time has at itself. We can express this more general idea using quantification over times, as follows:

NOW+ Necessarily, at each time t, every proposition is, necessarily, true just in case it is true at t.
$\Box \forall t \, \mathrm{At}\ t \, \forall p \Box (p \leftrightarrow \mathrm{At}\ t\ p)$[21]

As Fine (1977: 169) puts it: 'At each time, the same present runs through each possible world'. The necessary permanent truth of *Supervenience* is an immediate consequence of NOW+: necessarily, at any time t, for any true proposition p, the proposition that p is true at t is a permanent truth that necessitates p.[22]

As defenders of *Perpetuity*, we of course think there are counterexamples to NOW. Consider the proposition that dinosaurs roam the Earth. It is not true, so it has never been *now* true. But it has been true, so it has been true without being now true. So by *Perpetuity* it must be possible for it to be true without being now true. With such propositions squarely in view, we find that *Perpetuity* remains compelling, and the plausibility of NOW correspondingly diminishes. Without some further argument, then, rejecting NOW doesn't strike us as particularly costly. But there are some interesting arguments for NOW. We will consider two of them.

First: NOW can be derived from the following claims: (a) 'necessarily' commutes with 'now' (i.e. the propositions that are now necessarily true are exactly those that are necessarily now true); (b) every proposition is necessarily now: true if and only if now true; and (c) every proposition that is now true is true. Given (a), (b) implies that every proposition is now necessarily: true if and only if now true. And given (c), we can delete the initial 'now' from this claim to derive NOW.

We see no convincing grounds to accept (a), on its intended interpretation as a claim about metaphysical necessity.[23] While 'necessarily now φ' and 'now necessarily φ' may be interchangeable in typical contexts, it is also clear that in typical contexts, prefixing a context-sensitive sentence with 'now' will lead us to favor resolutions of its context-sensitivity on which it expresses a non-eternal proposition — otherwise, the 'now' would be pointless. But given *Perpetuity* and modal S5, all attributions of metaphysical necessity are eternal: if true they are necessarily true, and hence always true; if false they are necessarily false, and hence always false. So in typical contexts it will not be natural to interpret the 'necessarily' in 'now necessarily φ' as expressing metaphysical necessity. For this reason, one should be wary of drawing any conclusions about metaphysical necessity from our intuitive reactions to the schema 'Necessarily now φ just in case now necessarily φ'.[24]

Second: NOW can be derived from certain premises about counterfactuals: (i) Every proposition would have been true now if it had been true; (ii) No possibly true proposition is such that a contradiction would have been true if it had been true; (iii) 'Now' commutes with truth-functional connectives (even under counterfactual suppositions); (iv) 'Now now' is intersubstitutable with 'now' (even under counterfactual suppositions). (*Proof*: Let $>$ abbreviate the counterfactual conditional. By (i), $\forall p(\neg(p \leftrightarrow \mathrm{N}p) > \mathrm{N}\neg(p \leftrightarrow \mathrm{N}p))$. By (iii) this is equivalent to $\forall p(\neg(p \leftrightarrow \mathrm{N}p) > \neg(\mathrm{N}p \leftrightarrow \mathrm{NN}p))$, and hence, given (iv), to $\forall p(\neg(p \leftrightarrow \mathrm{N}p) > \neg(\mathrm{N}p \leftrightarrow \mathrm{N}p))$. But $\neg(\mathrm{N}p \leftrightarrow \mathrm{N}p)$ is a contradiction, so by (ii), $\forall p\Box(p \leftrightarrow \mathrm{N}p)$.)[25]

To resist this argument, proponents of *Perpetuity* must deny one of (i)–(iv). We will focus on (i), although we also have some doubts about (ii) and (iii). Granted, it generally seems fine to insert and delete 'now's in the consequents of ordinary counterfactuals. But since most of the counterfactuals we ordinarily consider have antecedents which could have been true now, it is tendentious to abstract from this pattern a rule which licenses such insertions and deletions even in counterfactuals whose antecedents could not possibly have been true now. For example, 'now' seems not to be redundant in 'If we were at a philosophy conference in the year 2500, we would probably not be talking about any issue now regarded as important.' Analogously, in evaluating ordinary counterfactuals we freely help ourselves to the actual laws of nature (see Goodman 2015 and Dorr 2016), but it would be very tendentious to extract from this practice a general principle to the effect that the actual laws would have been true no matter what.[26]

V

A second argument for *Symmetry* appeals to the following two premises:

ACT Every proposition is, always, true if and only if actually true.
$\forall p\ \mathrm{A}(p \leftrightarrow @p)$

$\mathrm{RIG}_@$ Every actually true proposition is necessarily actually true.
$\forall p(@p \rightarrow \Box @p)$

For the same reason that NOW and $\mathrm{RIG_N}$ jointly imply *Supervenience*, ACT and $\mathrm{RIG}_@$ jointly imply *Supervenience** (the result of interchanging temporal and modal operators in *Supervenience*), and hence also imply *Supervenience* and *Symmetry* (which we showed to be equivalent to *Supervenience** in §III).[27] Assuming propositional temporalism, they also can be used to generate explicit counterexamples to *Perpetuity*: whenever p is temporarily true, the proposition that p is actually true is necessarily true, by ACT and $\mathrm{RIG}_@$, and yet sometimes false, by ACT.

We do not deny that both (the English versions of) ACT and $\mathrm{RIG}_@$ have readings on which they express obvious truths.[28] But English sentences involving 'actually' often have several readings, and we see little reason to think there is any non-equivocating reading of ACT and $\mathrm{RIG}_@$ on which both are true.[29] The acceptability of sentences like 'I could have actually run you over!' shows that adding 'actually' sometimes makes no discernible difference to truth conditions. Read in the corresponding way, the following principle is true:

$\mathrm{ACT}_\Box$ Every proposition is, necessarily, true if and only actually true.

But on the interpretation corresponding to this one, $\mathrm{RIG}_@$ is false, since together with $\mathrm{ACT}_\Box$ it would entail that every truth is a necessary truth.

Our opponents might object to our assimilation of ACT to $\mathrm{ACT}_\Box$ by pointing to a contrast between the ways 'actually' embeds in temporal and modal environments. Consider:

(1) a. The climate could be warmer than it actually is.
b. The climate will be warmer than it actually is.

(1a) is fine, whereas (1b) sounds odd, at least out of the blue.[30] (Our informants said things like 'That is not the right way to say it in English: you should say "now" rather than "actually"'.) So our opponents might argue as follows: (1b) is infelicitous because it is false on all readings; the falsity of (1b) is best explained by a more general principle from which it follows that ACT is *true* on all readings; hence, since it is agreed that $\mathrm{RIG}_@$ is true on at least one reading, it follows that there is at least one reading on which both are true, so that the argument against *Perpetuity* goes through.

We think that this argument fails at the first step (which is not to concede that the rest of the argument is unproblematic). Although there is a contrast between (1a) and (1b) as regards the ease of accessing true readings, with the right setup — for example, when a certain fiction, misapprehension, or only recently ruled out hypothesis about the current climate is salient — (1b) can sound fine. There are also theoretical reasons to think that (1b) has a true reading. Consider the results of deleting 'actually' from (1a) and (1b):

(2) a. The climate could be warmer than it is.
b. The climate will be warmer than it is.

(2b) clearly has a true reading, and in view of the equivalence of (2a) and (1a), it is hard to see how the addition of 'actually' in (1b) could prevent it from having a parallel reading.[31]

The infelicity of (1b) discourse initially is an instance of a general feature of 'actually' that has nothing to do with temporal environments. In response to the question 'What do you do?', it would be odd to reply 'Actually I am a doctor'. No doubt this is because the usual role of 'actually' is to signal some kind of surprise or contrast, or that something is being corrected, or the like. When it is not clear from context how it could play any of these roles, it is typically infelicitous. In view of this generalization, the striking fact is that (1a) *is* felicitous out of the blue, or, more generally, that 'actually' embeds more happily under certain modals than elsewhere. But we will not speculate about the explanation of this fact, since we see no reason to think that it bears on the question of this paper.[32]

One might reply that even if the use of @ that makes $\mathrm{RIG}_{@}$ unambiguously true is a philosopher's invention, it is intelligible and useful. We agree. But clearly we have no business having pre-theoretic judgments about the truth of ACT when @ is introduced in part by the stipulation that $\mathrm{RIG}_{@}$ is true.

Another way to introduce @ as a piece of philosopher's jargon is to appeal explicitly to the metaphysics of possible worlds: we could introduce a name 'α' for the actual world, and let '$@p$' abbreviate 'p is true at α'. But on this interpretation, the right way to assess ACT and $\mathrm{RIG}_{@}$ is again not on the basis of their pre-theoretical plausibility, but as part of some broader theory about possible worlds. In the next section we consider how such theories bear on *Perpetuity*.

VI

Philosophers writing about possible worlds often take the following principles for granted:

Leibnizian Possibility: A proposition is possibly true if and only if it is true at some possible world.

$$\Diamond p \leftrightarrow \exists w(\mathrm{At}\ w\ p)$$

Conjunction: The conjunction of two propositions is true at a possible world if and only if both of those propositions are true at that world.

$$\mathrm{At}\ w(p \wedge q)) \leftrightarrow (\mathrm{At}\ w\ p \wedge \mathrm{At}\ w\ q)$$

Negation: The negation of a proposition is true at a possible world if and only if that proposition is not true at that world.

$$\mathrm{At}\ w\, \neg p \leftrightarrow \neg \mathrm{At}\ w\ p$$

Historicity: Possible worlds that agree about how things always are agree about everything.

$$\forall p\, \big(\mathrm{At}\ w\ \mathrm{A}p \leftrightarrow \mathrm{At}\ w'\ \mathrm{A}p\big) \rightarrow \forall p(\mathrm{At}\ w\ p \leftrightarrow \mathrm{At}\ w'\ p)$$

These principles jointly imply *Symmetry*, given a further principle that is valid in the background logic:

Encylopedia: Some permanent truth necessitates every permanent truth.

$$\exists q(\mathrm{A}q \wedge \forall p(\mathrm{A}p \to \Box(q \to p)))$$

(*Proof*: We will derive *Supervenience*, which is equivalent to *Symmetry*. Let h witness the truth of *Encylopedia*. We first show that any two worlds w and w' at which h is true agree about everything. Let p be any proposition. If p is always true, then the proposition that p is always true is itself always true, and thus necessitated by h. So $h \wedge \neg\mathrm{A}p$ is not possible; by *Leibnizian Possibility* it is true at no world; so by *Conjunction* and *Negation*, $\mathrm{A}p$ is true at every world where h is true, and in particular at w and at w'. If on the other hand p is not always true, then the proposition that p is not always true is always true, and thus necessitated by h. So $h \wedge \mathrm{A}p$ is not possible; by *Leibnizian Possibility* it is true at no world; so by *Conjunction*, $\mathrm{A}p$ is neither true at w nor true at w'. So for any p, $\mathrm{A}p$ is true at both or neither of w and w', which by *Historicity* implies that w and w' agree about everything. Next, to establish *Supervenience*, suppose p is true. Then $p \wedge h$ is true, and therefore possible, and therefore true at some world w by *Leibnizian Possibility*. By *Conjunction*, h and p are both true at w, so by the result just established, p is true at every possible world at which h is true. So by *Negation* and *Conjunction*, there is no possible world at which $h \wedge \neg p$ is true, and hence by *Leibnizian Possibility* $h \wedge \neg p$ is not possible: p is necessitated by the permanent truth h.)

Since propositional eternalism is a common presupposition of theorizing about possible worlds, it is worth checking that the four principles above are compatible with propositional temporalism. This can be seen by observing that, unlike propositional eternalism, all four principles are valid on the class of product models from §II. In combination with propositional temporalism, these principles suggest a "changing pluriverse" way of thinking about time and modality. A proposition is metaphysically possible just in case it is true at some world. But the facts about what is true at which worlds are temporary, so some of the propositions that are possible today will be impossible tomorrow.[33]

Like many other metaphysicians, we think it is dangerous to let one's opinions about modal questions be driven by one's theory of possible worlds, rather than the other way around. So we do not think it is much of a count against the conjunction of *Perpetuity* and propositional temporalism that it requires giving up at least one of *Leibnizian Possibility*, *Conjunction*, *Negation*, and *Historicity*.

How should friends of *Perpetuity* and propositional temporalism think about worlds and truth at a world? Perhaps the simplest option is to give up *Historicity*, keep the other three principles, and think of possible worlds as "points in logical space", corresponding to propositions p which are possibly modally maximal truths, in the sense that $\Diamond(p \wedge \forall q(q \to \Box(p \to q)))$.[34]

Alternatively, we could preserve *Historicity*, thinking of possible worlds as "possible histories", corresponding to propositions p which are possibly modally maximal *permanent* truths: $\Diamond(\mathrm{A}p \wedge \forall q(\mathrm{A}q \to \Box(p \to q)))$. When, in the course of a possible history, a proposition is sometimes true and sometimes false, the world is not enough to settle its truth value.[35] To figure out which of the other three principles fails, we need to decide what it means for a proposition to be true "at" a world. There are two obvious options. The first holds that p is true at w just in case p is necessitated by the world-history corresponding to w. This preserves *Conjunction* but requires giving up *Negation* and *Leibnizian Possibility*.[36] This view also rejects the following principle:

> *Disjunction*: The disjunction of two propositions is true at a possible world if and only if one of those propositions is true at that world.
>
> $$\text{At } w(p \vee q) \leftrightarrow (\text{At } w\ p \vee \text{At } w\ q)$$

The second option holds that p is true at w just in case p is compossible with the world-history corresponding to w. On this approach, *Leibnizian Possibility* and *Disjunction* hold, but *Conjunction* and *Negation* fail.[37] Moreover, for any p, $p \vee \mathrm{A}\neg p$ will be compossible with every world-history despite being contingent, and will thus be a counterexample to:

> *Leibnizian Necessity*: A proposition is necessarily true if and only if it is true at every possible world.
>
> $$\Box p \leftrightarrow \forall w(\text{At } w\ p)$$

On both the "pointy" and "historic" conceptions, worlds correspond to certain propositions, only one of which is true. Call this world 'α' ('the actual world') and interpret '@' as synonymous with 'At α'. Assuming *Perpetuity* and propositional temporalism, we know from the previous section that at least one of ACT and $\text{RIG}_@$ must fail on this interpretation. In fact, since the all the views we have considered make facts about what is true at a world non-contingent, they all preserve $\text{RIG}_@$ and thus reject ACT. On the pointy conception, and on the historic conception with the compossibility interpretation of 'At', any temporarily true p is a counterexample to ACT: $@p$ is necessarily true, hence always true, so $p \leftrightarrow @p$ is only temporarily true. On the historic conception with the necessitating interpretation of 'At', any temporarily false p is a counterexample to ACT: $@p$ is necessarily false, hence always false, so again $p \leftrightarrow @p$ is only temporarily true.

Our attitude towards the argument for *Symmetry* from ACT and $\text{RIG}_@$ on the world-theoretic interpretation is the same as our attitude to the argument considered earlier in this section: 'possible world' is enough of a term of art that we are unperturbed by the prospect of giving up one of these two principles.

Indeed, in view of the potential for equivocation between pointy and historic conceptions of worlds, we think it best to avoid 'world'-talk altogether in theorizing about temporary matters. But if forced to choose, we would favor the pointy conception, since *Leibnizian Possibility* and *Leibnizian Necessity* seem particularly central for the usual applications of world-talk. Moreover there are three further reasons to dislike the historic conception, at least when combined with a necessitating or compossibility interpretation of truth at a world: first, it seems arbitrary to choose between *Leibnizian Possibility* and *Leibnizian Necessity*; second, it seems arbitrary to choose between *Conjunction* and *Disjunction*; and third, the failure of truth to be coextensive with truth at the actual world runs completely counter to the way philosophers are used to talking about worlds.[38]

VII

We turn finally to a more abstract and theoretical strategy for arguing for *Symmetry.* Consider the following schematic premises:

F-Supervenience: Every truth is necessitated by some type-F truth.

$$\forall p(p \rightarrow \exists q(q \wedge \mathrm{F}q \wedge \Box(q \rightarrow p))$$

F-Eternalism: Every type-F truth is always true.

$$\forall p((p \wedge \mathrm{F}p) \rightarrow \mathrm{A}p)$$

Together these premises immediately imply *Supervenience*, and hence *Symmetry*. Moreover, there are various interpretations of 'type-F propositions' on which both premises of this argument have some plausibility, such as propositions about how things are in "metaphysically fundamental" respects, propositions about microphysics, and (of particular interest) spacetime-theoretic propositions. Since Einstein and Minkowski, we have learnt to characterize the physical world in terms of the distribution of field-values in four-dimensional spacetime, rather than the evolution of field-values in three-dimensional space. It is natural to think that there is already something temporal about points of space*time*, so that it makes no sense to suppose, for example, that a particular spacetime point will change from having a low mass-density to having a high mass-density, just as it would make no sense to suppose that a particular instant of time will change from being one at which it is raining to being one at which it is not raining. And given a broadly physicalistic outlook, the success of this way of doing physics supports the thesis that all truths supervene on truths about the distribution of physical fields in spacetime, perhaps by way of the thought that only such truths are fundamental and all truths supervene on the fundamental truths.[39]

This strikes us as an important line of argument. But it also looks to us like an argument for propositional eternalism. It is hard to see how it could persuade a propositional temporalist to accept *Symmetry*, since it is unclear what could motivate *F-Supervenience* without also motivating the stronger thesis that for every truth there is a type-F truth that *always* necessitates it. One obviously cannot appeal to the claim that every truth *is* a type-F truth. One might try appealing to the thesis that every truth is in some sense "determined" or "grounded" by a type-F truth. But if grounding is simply identified with necessitation, this is question-begging, while if it is understood in some other way, it is hard to see why grounded truths should be necessitated by the truths that ground them, given that they need not be eternally implied by the truths that ground them.

F-Supervenience and *F-Eternalism* have further puzzling consequences on the assumption that there are temporarily true qualitative propositions.[40] Let an

F-possibility be a possibly-true proposition that conjoins *F-Supervenience* with a maximally specific type-F proposition. Assuming that being type-F is a necessary property of propositions, every F-possibility necessitates every proposition with which it is compossible.[41] For example, each F-possibility either necessitates that the number of stars is odd, or necessitates that it is not odd. But if there is qualitative change, there will presumably be many F-possibilities that necessitate that the number of stars is sometimes but not always odd. The division of these F-possibilities into those that necessitate that the number of stars is odd and those that necessitate that it isn't odd seems like it must exhibit a certain arbitrariness that we should hope to avoid when such purported necessary connections are at issue.

One might try to mitigate such arbitrariness by postulating that there is a certain three-dimensional slice *s* through spacetime, such that what propositions an F-possibility necessitates is a function of what it says about *s*. For example, perhaps an F-possibility necessitates that the number of stars is odd when it entails that *s* contains an odd number of appropriately shaped, high-temperature subregions. (Presumably this proposal will be combined with the claim that which region of spacetime is modally distinguished in this way is constantly changing.) But this suggestion introduces new problems. First, in distinguishing one slice through spacetime as special in this way, the proposal conflicts with what is widely taken to be a basic moral of relativity physics.[42] Second, it is in tension with the following claims: (i) if *F-Supervenience* is true, then so too is the analogous thesis restricted to purely qualitative propositions, namely that every qualitative truth is necessitated by a qualitative spacetime-theoretic proposition; (ii) there are pairs of F-possibilities that agree on all qualitative spacetime-theoretic propositions but disagree as regards the qualitative spacetime-theoretic role played by *s*.[43] For example, if two such F-possibilities disagree as regards whether *s* contains an odd number of high-temperature star-shaped regions, the proposed special status for *s* will require them to necessitate incompatible qualitative propositions, in violation of (i). Thirdly, the proposal breaks down when we consider metaphysical possibilities in which *s* is not part of the spacetime manifold at all: for example, possibilities where spacetime undergoes an early gravitational collapse. (We will have more to say about such possibilities in §IX.)

VIII

Having addressed a range of arguments for *Symmetry*, in this section and the next we will go on the offensive and develop two arguments against *Symmetry*. The basic thought behind both arguments is that *it is contingent what times there are*. This is a plausible idea. Moreover, its plausibility is independent of the plausibility of *Perpetuity*, and so it is dialectically appropriate in the context of a defence of *Perpetuity*. Indeed Fine, immediately after presenting the product model theory, suggests that the account is 'perhaps over-simple', and that a 'more sophisticated' account would allow for contingency as regards what times there are (Fine 1977: 167).

Our first argument turns on there being contingency as regards the *cardinality* of the time series. A nice feature of this argument is that it can be formalized in a way that does not explicitly talk about times at all, using instead only tense operators and propositional quantifiers. For any natural number n, the claim that the time-series has a cardinality of at least n can be formalized as the claim that there are n propositions, each of which is sometimes true, and no two of which are ever both true. And although our basic language $\mathcal{L}$ does not contain the resources to distinguish different infinite cardinalities that the time-series might have, there are several natural extensions of $\mathcal{L}$ which do allow such distinctions to be drawn. For example, we could add higher-order quantifiers, e.g. into the position of n-ary sentential operators, which would allow us to adapt standard set-theoretic ways of characterizing different infinite cardinalities. Or we could move to an infinitary language in which the analogues of the above finite quantified claim could be expressed directly. Or we could simply add the standard future and past tense operators G and H, which make it possible to formulate various claims about the before-after structure of the time-series which (intuitively and model-theoretically) have cardinality-theoretic implications, although they are strictly stronger than any mere cardinality claim. For example, using these tense operators together with propositional quantifiers, we can say that the time-series is a discrete, dense, or continuous linear ordering.[44]

We maintain that such cardinality-theoretic claims are metaphysically contingent. For example, it is metaphysically possible for time to be structured either like the integers, like the rationals, or like the real numbers. This judgment is bolstered by the judgment that the structure of *spacetime* is contingent in parallel ways: spacetime could have been continuous, discrete, dense but countable, etc. Admittedly, such possibility claims are controversial: they will be denied, for example, by those who identify metaphysical possibility with physical possibility. Moreover, it is a vexed question exactly what the connection is between the metaphysics of spacetime and the metaphysics of time. But it would be extremely surprising if there were *no* such connection: it is not a mere coincidence that spacetime and time are both continuous (or both discrete, or both dense-but-countable, as the case may be).

In addition to contingency in the cardinality of the time-series, our argument will require two other premises. The first — which we don't expect to be controversial — is that if *Symmetry* is true, then it is necessarily always true. The second is that *metaphysical modality is not a source of temporariness in its own right*. The thought is that the attribution of metaphysical possibility or necessity to an eternal proposition results in another eternal proposition. Intuitively, an eternal proposition is one that is non-temporary not just *de facto* but of its nature. However this status is understood, clearly Ap should count as eternal for any p, and clearly $\Box\mathrm{A}(p \to \mathrm{A}p)$ should be true for any eternal p. Thus, if the underlying thought about eternalness is true under any reasonable interpretation, so is

> *Eternity*: Necessarily always: if a proposition is necessarily always true, it is always necessarily always true.
>
> $$\forall p(\Box\mathrm{A}(\Box\mathrm{A}p \to \mathrm{A}\Box\mathrm{A}p))$$

We think that *Eternity* is attractive in its own right, and not merely because it is a consequence of (the necessary permanent truth of) *Perpetuity*. For example, *Eternity* will appeal to those attracted to the idea that eternal truths are "accessible from a God's eye point of view" in a way that temporary truths are not, since one would expect that if the truth of a proposition is open to God's view, then so too is its modal status.[45]

To see why *Eternity* and the necessary eternal truth of *Symmetry* are inconsistent with contingency in the cardinality of the time series, first note that that *Eternity* logically implies that the *Church-Rosser* principle from §III — $\forall p(\Diamond \mathrm{A}p \rightarrow \mathrm{A}\Diamond p)$ — is true necessarily and always. (*Proof*: suppose that $\Diamond \mathrm{A}p$. Then by temporal S5, $\mathrm{AS}\Diamond\mathrm{SA}p$, i.e. $\mathrm{A}\neg\mathrm{A}\Box\mathrm{A}\neg\mathrm{A}p$. So by the contrapositive form of *Eternity,* $\mathrm{A}\neg\Box\mathrm{A}\neg\mathrm{A}p$, i.e. $\mathrm{A}\Diamond\mathrm{SA}p$, which implies $\mathrm{A}\Diamond p$ by temporal S5.[46]) Given this result, the inconsistency we are interested in follows from a result proved in §III, namely that any relational structure in which *Symmetry* is necessarily always true and *Church-Rosser* is both necessarily and always true is isomorphic to a product structure. In any such structure, all histories have the same cardinality, so any sentence entailing contingency in the cardinality of the time series must be false.[47]

As noted above, our simple language lacks $\mathcal{L}$ the resources to say that the cardinality of the time series is infinite, or to distinguish between different possible infinite cardinalities of the time series. But it does allow us to talk about finite cardinalities, and since some might want to escape the present objection to *Symmetry* by rejecting our background logic, it may be instructive to consider object-language arguments from *Eternity* and (the necessary permanent truth of) *Symmetry* to the falsehood of particular claims to the effect that there are n times and could have been more or fewer than n times. An especially easy case is that of cardinality 1. Insofar as the general idea of contingency in the cardinality of the time series is well-motivated, it is hard to deny that, as a limiting case, there could have been *exactly one* time: i.e. that it could have been that everything true was always true ($\Diamond\forall p(p \rightarrow \mathrm{A}p)$). But given *Eternity*, this claim directly implies *Perpetuity*, and is thus inconsistent with the combination of *Symmetry* and propositional temporalism. For suppose $\Box q$. Then $\Box\Box q$; so $\Diamond(\Box q \wedge \forall p(p \rightarrow \mathrm{A}p))$, so $\Diamond\mathrm{A}\Box q$. By *Eternity* this yields $\mathrm{A}\Diamond\mathrm{A}\Box q$; but this implies $\mathrm{A}\Diamond\Box q$ (by the factivity of A) and hence $\mathrm{A}q$ (by the modal B schema). Similar arguments can be given for finite cardinalities greater than one.[48]

In summary, proponents of *Symmetry* face a choice. They can accept *Eternity*, and deny that it is contingent how many times there are. Or they can reject *Eternity*, and hold that metaphysical possibility and necessity are sources of temporariness in their own right. The implausibility of both options constitutes an argument against *Symmetry* that is independent of judgments about *Perpetuity*.

IX

Let us now turn our attention to contingency in the *composition* of the time series — i.e. which particular times are ever present. There are three possible views:

Tomorrow Never Dies: Every time is necessarily sometimes present.

$$\forall t(\Box \mathrm{S}\ \mathrm{Present}(t))$$

Die Another Day: Some times, but not the present time, are possibly never present.

$$\exists t(\Diamond \neg \mathrm{S}\ \mathrm{Present}(t)) \wedge \forall t(\mathrm{Present}(t) \rightarrow \Box \mathrm{S}\ \mathrm{Present}(t))$$

Live And Let Die: The present time is possibly never present.

$$\exists t(\mathrm{Present}(t) \wedge \Diamond \neg \mathrm{S}\ \mathrm{Present}(t))$$

In this section we will first argue against *Tomorrow Never Dies* and *Die Another Day.* We will then argue that if *Live and Let Die* is true, NOW (discussed in §V) must be rejected, thereby undermining the most intuitive argument for *Symmetry.*

The previous section discussed one good reason to reject *Tomorrow Never Dies*, namely that it is possible that the time-series has a smaller cardinality than it in fact has. But there are other arguments against *Tomorrow Never Dies* that don't turn on that possibility, and which can also easily be turned into arguments against *Die Another Day*. First, one might argue from the premise that there could have been a cosmic catastrophe in which history itself came to an end, so that some times which are in fact a proper initial segment of the time series would instead have been the totality of the time series. For example: should the sky fall tomorrow, times that will in fact be present next week would never get to be present. This argument generalizes to an argument against *Die Another Day*, assuming that if such a cosmic catastrophe is possible at all, then there could have been such a catastrophe some time in the past, so that the present time would have been absent from the time series. Second, one might appeal to the idea that the temporal relations between times are essential to them, in the sense that necessarily, if t is present before t', then it is not possible that t or t' ever be present without t being present before t'. Combined with the claim that the before-after structure of the time series could have been different — e.g. because it is contingent whether time is circular, or whether there is a first or last moment — this essentialist thesis implies that every time is possibly never present, thus ruling out both *Tomorrow Never Dies* and *Die Another Day.* (Note that the premises of these two arguments are incompatible, since the essentialist premise rules out the possibility of the time-series being a proper initial segment of the actual time-series.) A third kind of argument turns on theses about the relation of time to spacetime. For example, one might think that for every time t there is a region of spacetime s_t such that necessarily, t is part of the time series just in case s_t is an appropriate kind of part (a "simultaneity slice") of the spacetime manifold. (One version of this view identifies each time t with the corresponding region s_t.) Contingency in the composition of the time series then follows from contingency in the composition of the spacetime manifold: *Tomorrow Never Dies* is ruled out if some time corresponds to a region which could have failed to be a simultaneity slice, and *Die Another Day* is ruled out if the present time corresponds to such a region.[49] Note that all three of these arguments are compatible with the necessity of the cardinality of the time-series, as well as with the necessity of the laws of physics.[50]

Those who want to maintain *Tomorrow Never Dies* in the face of these arguments might offer a rejoinder inspired by Timothy Williamson's defense of the claim that it is necessary what things there are (Williamson 2013). According to Williamson, there are a great many very boring things in addition to the interesting ones with which we are familiar. Since Wittgenstein could have had a child, there are things that could have been children of Wittgenstein; but each is a mere spectre: it is not a person, and has no mass, has no spatial location, etc. Williamson thinks that his opponents are, by and large, right about the ways in which it is contingent what interesting things there are, but fall into error by failing to take boring things into account. Defenders of *Tomorrow Never Dies* might analogously suggest that we have fallen into error by confusing the true claim that it is a contingent matter which times are *interesting* with the false claim that it is a contingent matter which times are sometimes present. "Boring" times could then be characterized as times at which all *objects* are boring, in the sense in which Wittgenstein's possible children are boring on Williamson's view.

Even those who are happy with Williamson's defense of necessitism have reasons to be cautious about this strategy. Just as Williamson counters the claim that there could have been things that are not actually anything by saying that there actually *are* boring things, so too proponents of the "boring times" strategy will counter our claim that there could have been times that are actually never present by saying that there actually are *sometimes-present* boring times (and indeed, infinitely many of them). In other words: sometimes, nothing is interesting. But isn't the question whether there have always been, and always will be, interesting things (such as electrons) a question for physics, not to be answered by philosophers from the armchair? Furthermore, whereas Williamson can and does deny that boring objects bear nontrivial spatial relations to interesting ones or to one another, proponents of the "boring times" strategy cannot likewise deny that boring times bear nontrivial temporal relations to the interesting times in which we live, or to one another — at least holding fixed the truisms that anything that is sometimes true either is, was, or will be true, and that no two times are ever both present at once. So: have you been interesting ever since you were born? Are the boring times clustered at one or other end of the time series? And how are boring times ordered — densely, discretely, continuously, or in some other way? These embarrassing questions suggest that the present strategy is unlikely to provide a solid dialectical basis for a defense of *Tomorrow Never Dies*.

Let us now return to *Die Another Day*. Even setting aside the arguments above, there is something bizarre about the proposal that some times are modally robust (necessarily sometimes present) in a way that other times are not. Moreover, on pain of arbitrariness, anyone who thinks that the present time is modally robust should accept the more general principle that every time is modally robust when it is present.[51] In combination with *Die Another Day*, this general principle entails that modal robustness is a temporary feature of times. And this consequence is inconsistent with *Eternity*, which we defended in §VIII. That principle says that metaphysical necessity is not itself a source of temporariness — necessitation never turns an eternal proposition into a temporary one. But according to the view under consideration, each time t that is only contingently ever present generates a counterexample to *Eternity*, since although the proposition that t is sometimes present is eternal, its necessitation is temporary (false now, but true when t is present).

Having just argued for *Live And Let Die* by arguing against its two alternatives, we will now argue that it implies the falsity of NOW, as follows:

(1) The present time is possibly never present.

$$\exists t(\text{Present}(t) \wedge \Diamond\neg S\,\text{Present}(t))$$

(2) So the present time is not necessarily present.

$$\exists t(\text{Present}(t) \wedge \neg\Box\text{Present}(t))$$

(3) So it is not necessary that all and only true propositions are true at the present time.

$$\exists t(\text{Present}(t) \wedge \neg\Box\forall p(p \leftrightarrow \text{At } t\ p))$$

(4) So not every proposition is, necessarily, true just in case true at the present time.

$$\exists t(\text{Present}(t) \wedge \neg\forall p\Box(p \leftrightarrow \text{At } t\ p))$$

(5) So not every proposition is, necessarily, true just in case now true.

$$\neg\forall p\Box(p \leftrightarrow \text{N}p)$$

(1) is *Live and Let Die*; (5) is the negation of NOW. We will discuss each step in turn.

From (1) to (2): The validity of this inference follows from the plausible claim that for something to be never true just is for it to neither be true, have been true, nor be going to be true. Thus nothing could possibly be both present and never present. If you disagree with us about this, we can without any loss of plausibility replace *Live and Let Die* with the claim that the present time could have been neither present, formerly present, nor ever going to be present, and never say 'never' again.

From (2) to (3): Say that t is *accurate* just in case all and only the true propositions are true at t. (2) implies (3) so long as being accurate necessitates being present. This is true on all of the most natural accounts of the connection between 'present' and 'At t':

(a) To be present is to be accurate.
(b) For p to be true at t is for it always to be the case that if t is present, p is true.
(c) For p to be true at t is for it sometimes to be the case that t is present and p is true.
(d) For p to be true at t is for it to be the case that p would be true if t were present.[52]

For (a), the implication from accuracy to presence is immediate. For (b) and (d), it follows from the fact that the proposition that t is present cannot fail to be true at t, so if t is accurate this proposition must be true. For (c), it follows from the fact that the proposition that t is not present cannot be true at t, so if t is accurate this proposition must be false.

Are there any principled views about the connection between 'present' and 'at t' that could allow for the possibility of accuracy without presence? One might suggest a view on which things that are never present get to count as accurate 'by courtesy', as in the following variant of (c):

(e) For p to be true at t is for it to be the case that either it is sometimes the case that (p is true and t is present), or p is true and t is never present.

This view blocks the implication from accuracy to presence and thus from (2) to (3). The problem with it is that it is incompatible with the principle that what is true at t is an eternal matter: $\forall t \Box \forall p(\text{At } t\, p \to \text{A At } t\, p)$. For presumably, if the present time could have been never present, then propositional temporalism could still have been true in such a possibility: that is, there could have been some temporarily true proposition. (e) then entails that the present time is such there could have been a proposition that was true at it but not always true at it. This is bizarre: how could it have been sometimes raining now and sometimes not raining now?[53,54]

From (3) to (4): The validity of this inference follows from the validity of the modal Barcan Formula, $\forall p \Box \varphi \to \Box \forall p \varphi$. Those who think that there could have been propositions in addition to those there actually are might thus consider resisting at this step. They might think that the present time is necessarily accurate with regard to all the propositions there actually are (as required for (4) to be false) while maintaining that there could have been new propositions whose truth values differed from their truth values at the present time (which suffices for (3) to be true). However, there is a different way of arguing from (3) to (4), relying not on the Barcan Formula but instead on two hard-to-deny principles about the logic of 'At t': (i) 'At t' is closed under classical consequence; (ii) 'At t' is redundant when it occurs within the scope of 'At t' without any intervening modal or temporal operators. (Both principles are consequences of the analyses (b)–(e) of 'At t' in terms of 'Present' considered above.) Let t be the present time, and let q be the proposition that t is accurate. Necessarily, if anything is true at t, $\forall p(p \leftrightarrow p)$ is true at t by (i), in which case $\forall p(p \leftrightarrow \text{At } t\, p)$, i.e. q, is also true at t by (ii). Now suppose for contradiction that (3) is true and (4) is false. By (3), $\neg q$ is possibly true. And by the negation of (4), every proposition (including $\neg q$) is, necessarily, true just in case true at t. Hence it is possible that $\neg q$ is true at t. But by the earlier result, it is necessary that q is true at t if anything is. So by (i), it is possible that $q \wedge \neg q$ is true at t; hence by the falsehood of (4), it is possible that $q \wedge \neg q$: contradiction. We thus see no plausible way to resist the step from (3) to (4).

From (4) to (5): This step should be uncontroversial. (Some might reject the *equivalence* of (4) and (5) on the grounds that that there are no times, but even they will accept the relevant material implication.)

This concludes our argument against NOW. It is addressed, in the first instance, to those who, when confronted with the tension between NOW and *Perpetuity*, were initially inclined to find NOW the more plausible principle.[55] But the falsity of NOW also causes trouble for those who accept *Symmetry* on some other grounds, and thus reject *Perpetuity*. Consider the question: which possibly sometimes-true propositions are possibly true? According to proponents of *Perpetuity*: all of them. According to proponents of NOW: those that are possibly true *now*. But what about people who reject both *Perpetuity* and NOW? They must think that some but not all propositions that could have been sometimes true but could not have been true now are possibly true. If they accept (the necessary eternal truth of) *Symmetry,* they must also think that, for each possible world-history, there is a unique time in that history which could have been present had that history obtained. Given the falsity of NOW, this function from possible world-histories to members of their respective time-series cannot be the constant function that maps every history to the present time. But it is hard to see how this function could then fail to draw arbitrary distinctions of a sort that ought to disqualify it from marking the boundaries of metaphysical possibility.

X

Someone might accept everything we have said up to this point while nevertheless regarding the dispute we have been engaged in as "merely verbal". Perhaps our opponents mean something different from us by the term of art 'metaphysically necessary', such that (a) 'Every metaphysically necessary truth is always true' is false as used by them, and (b) both our way of talking and their' way of talking are "equally good". In particular, one might suggest that our opponents use 'metaphysically necessary' to express the notion of *immediate* necessity, where this can be defined in terms of *our* notion of metaphysical necessity as follows: it is immediately necessary that φ just in case the truth of φ is a metaphysically necessary consequence of the truth about which time is present. (In symbols: $\Box_{\mathrm{I}}\varphi =_{\mathrm{df}} \exists t(\mathrm{Present}(t) \wedge \Box(\mathrm{Present}(t) \rightarrow \varphi))$, where t is some time variable not free in φ). For any formula φ, let φ^{I} be the result of substituting immediate for metaphysical necessity in φ (i.e. the result of replacing each subformula of the form $\Box\psi$ with $\Box_{\mathrm{I}}\psi$). We can now express the interpretative hypothesis as follows: For any φ, φ as used by our opponents is equivalent to φ^{I} as used by us.

In favor of this hypothesis, one might appeal to the following facts:

(i) Any formula φ is logically equivalent to φ^{I} on the class of generalized product structures from §III, interpreting quantification over times as discussed in §II.
(ii) *Perpetuity*$^{\mathrm{I}}$ — $\forall p(\exists t(\mathrm{Present}(t) \wedge \Box(\mathrm{Present}(t) \rightarrow p)) \rightarrow \mathrm{A}p)$ — is uncontroversially false assuming propositional temporalism, since it entails that the present time is always present.
(iii) *Symmetry*$^{\mathrm{I}}$ — $\forall p(\neg p \rightarrow \exists q(\exists t(\mathrm{Present}(t) \wedge \Box((\mathrm{Present}(t) \wedge p) \rightarrow q) \wedge \mathrm{A}(p \rightarrow \neg q)))$ — is uncontroversially true, since the existential generalization is witnessed by the proposition attributing presentness to the present time.

(iv) NOW^I is uncontroversially true, since it is uncontroversial that the present time is such that necessarily, if it is present, then the true propositions are exactly those propositions that are true now.

Since our opponents think that *Symmetry* is not only true but necessarily true, necessarily always true, and so on, (i) makes it natural to think of them as recognizing only one notion of necessity where we recognize two. It might therefore be thought that we face an interpretative dilemma that should be resolved by considerations of charity. And given (ii)–(iv), such considerations seem to support the present interpretative hypothesis.

We are unmoved by this mode of argument. Philosophers do regularly make mistakes about general metaphysical principles, and equate statuses that are in fact distinct, so the fact that a certain interpretation avoids attributing such errors is very weak evidence that the interpretation is correct. But even if we were convinced that there was a practice afoot of using ‘metaphysically necessary’ to express immediate necessity, we would still emphatically reject the claim that this way of speaking was “just as good” as ours. For there are hypotheses about the possible structures of time that simply cannot be expressed in the language of tense operators, propositional quantifiers, and an operator expressing immediate necessity. For example, consider the following pair of hypotheses:

H1 It is metaphysically necessary that time is dense.
H2 Although it is only contingently true that time is dense, it is necessary that for each time t, either it is necessary that if t is ever present, time is dense, or else it is necessary that if t is ever present, time is not dense.

(Intuitively: whether a time belongs to a dense time-series is an essential property of it.) H1 and H2 strike us as perfectly respectable competing hypotheses about the modal metaphysics of time, about which there could be a substantive debate. But there seems to be no way of conducting such a debate using immediate necessity as one’s basic modal notion. For one thing, $H2^I$ is flatly inconsistent and so clearly fails as a way of making sense of H2. Nor would it help to instead replace ‘metaphysically necessary’ in H2 with ‘always immediately necessary’, or ‘immediately necessarily always’, or with any finite string of ‘always’ and ‘immediately necessarily’ operators, or even with the infinite conjunction of all such strings. We will focus on this last proposal. Model-theoretically, the effect of such a replacement is an operator corresponding to an accessibility relation which is the transitive closure of the union of the temporal accessibility relation and the “immediate possibility” accessibility relation (the relation one point bears to another if and only if is modally accessible and agrees about which time is present). Such an operator is not semantically equivalent to metaphysical necessity; the accessibility relation so-defined can fail to be universal even in a structure where modal accessibility is universal. In particular, H2 is only true in structures where this relation fails to be a universal relation: if H2 is true, there are possibilities where time is not dense, but they cannot be reached by any sequence of steps each of which either takes you to a different point in the same possible history or to a different possible history with the same present time.

By our lights, these expressive limitations constitute an important respect in which a language whose only modal operator is immediate necessity is inferior to ours. Moreover, this inferiority strikes us as a strong consideration against the hypothesis that our opponents are in fact speaking such a language.

To be clear, we are not advancing this "expressive power" consideration as an argument *for Perpetuity*. Our current target is the view that the debate over *Perpetuity* is merely verbal: we have argued that is not the case that both ways of speaking are equally good, on the grounds that if our way of speaking is good, our opponents' way of speaking is worse, since it is expressively impoverished relative to ours. We are not claiming that it is an important desideratum that a view be *neutral* as regards hypotheses like H1 and H2; on the contrary, we see settling such difficult questions as an advantage of a theory about time and modality. In metaphysics as in other areas of theory-building, strength is a virtue.

Our impulse to defend *Perpetuity* was initially triggered by incredulity that anyone would deny something so obvious. But our considered view is that this is an area of metaphysics where surprises may well be in store, and that competing theories should be adjudicated on broader theoretical grounds. This is why our defense of *Perpetuity* has taken the form of a critical investigation of opposing views — in particular, those that combine propositional temporalism with *Symmetry*. We have argued that such views are both under-motivated and incompatible with plausible claims about contingency in the time-series. The competing picture that accepts *Perpetuity* does not face these problems. Moreover, *Perpetuity* offers a simple and compelling explanation of its enormously many obviously and uncontroversially true instances. The balance of considerations thus tells firmly in its favor. No point in history lies outside the space of possibility. Diamonds are forever.[56]

Notes

[1] Assuming an S5 logic of necessity, *Perpetuity* is equivalent to the principle that what is possibly true is always possibly true — hence our title. *Proof*: Suppose p is possibly true; then it is necessarily possibly true (by the 5 axiom), and is therefore always possibly true (by *Perpetuity*). In the other direction, suppose p is necessarily true; then it is possibly necessarily true (by the T axiom), and is therefore always possibly necessarily true (by the titular principle). Since possible necessary truth entails truth (by the B axiom), it follows by the closure of 'always' under entailment that p is always true.

[2] See Prior 1971; Williamson 2003, §9; Williamson 2013, §5.9.

[3] Even those who harbor no doubts about the existence of propositions sometimes reject the intersubstitutability of φ and $\ulcorner$the proposition that φ is true$\urcorner$. For example, some think that if Socrates had not existed, then the proposition that Socrates does not exist would not have existed either, and so would not have been true. We will ignore such potential complications in what follows, since they do not, so far as we can tell, have any bearing on *Perpetuity*.

[4] This is not entirely obvious: some insist that "real" possibility requires historical possibility, where the only alternatives are "slippery subjective or linguistic or merely mathematical notions of possibility" (Belnap 2012: 7). Perhaps given these remarks, we should interpret these philosophers as holding that historical and metaphysical possibility coincide.

[5] Zalta (1988) also clearly embraces principles inconsistent with *Perpetuity*.

[6] See, e.g., Cresswell 1990; Kaufmann, Condoravdi, and Harizanov 2006; Schlenker 2006; and Rini and Cresswell 2012. More carefully: the views in question seem to entail that *if propositional temporalism is true* then *Perpetuity* is false. It is a vexed question whether or not such views take a stand on propositional temporalism (see King 2003; Ninan 2012; Schaffer 2012).

[7] We take it that in adopting a metaphysical interpretation of 'necessarily' philosophers are exploiting the normal context-sensitivity of the natural-language 'necessarily' rather than coining something altogether alien. This does not require thinking that the metaphysical interpretation is widespread outside of philosophy.

[8] It is not entirely unprecedented: the Hellenistic philosopher Diodorus Cronus argued that 'the possible is that which either is or will be', which entails (given a standard conception of necessity as dual to possibility) that the necessarily true propositions are exactly those that are true and always will be true, and hence include all the eternal truths; but he may not have been talking about metaphysical necessity. For an influential conjecture about Diodorus's argument see Prior 1955. During the medieval period it became customary to distinguish a notion of "possibility *per accidens*", conforming to Diodorus's definition, from other, less demanding notions of possibility — possibility *per se* and God's possibility — for which analogues of *Perpetuity* seem to have been accepted: see Knuutila 1982.

[9] $\mathcal{L}$ is in this respect simpler than the language of standard tense logic, in which $A\varphi$ is usually analyzed in terms of two other operators H ('it has always been the case that') and G ('it will always be the case that') as $H\varphi \wedge \varphi \wedge G\varphi$.

[10] It is important to distinguish accepting *Perpetuity* — a quantified sentence — from accepting all instances of the schema $\Box\varphi \to A\varphi$, since some parties to the debate accept *Perpetuity* without accepting the schema. For example, Salmon (1983, Appendix C) rejects the schema $\Box\varphi \to A\varphi$ but is a propositional eternalist and so certainly accepts *Perpetuity*. One way of making sense of such a position (although not Salmon's way) is to think of tense operators as binding unpronounced time variables, which deictically pick out particular times when they occur free. $\Box\varphi \wedge \neg A\varphi$ will then fail to entail $\exists p(\Box p \wedge \neg Ap)$ for the same reason that the consistent reading of 'He is happy but not every man is such that he is happy' fails to entail '$\exists p$(p but not every man is such that p)'.

[11] *Perpetuity* and propositional eternalism would still be equivalent if product models were enriched with a reflexive accessibility relation R on W and the clause for $\Box$ modified accordingly. More generally still, we can relativize R to members of T, defining $[\![\Box\varphi]\!]$ as $\{\langle w, t\rangle : \langle w', t\rangle \in [\![\varphi]\!]$ for all w' such that $wR_t w'\}$ (see Thomasson 1984).

[12] An alternative to explicit quantification over times is to enrich the language with devices of "temporal anaphora" (Kamp 1971, Vlach 1973, Cresswell 1990). We add to $\mathcal{L}$ countably many "time-storage" operators $\uparrow_0, \uparrow_1, \ldots$ and "then" operators $\downarrow_0, \downarrow_1 \ldots$. To interpret them, let assignment functions assign each "then" operator a member of T, and extend the interpretation function as follows:

$$[\![\downarrow_i\varphi]\!]^g = \left\{\langle w, t\rangle : \langle w, g(\downarrow_i)\rangle \in [\![\varphi]\!]^g\right\}$$
$$[\![\uparrow_i\varphi]\!]^g = \left\{\langle w, t\rangle : \langle w, t\rangle \in [\![\varphi]\!]^{g[\downarrow_i \mapsto t]}\right\}$$

where $g[\downarrow_i \mapsto t]$ is an assignment like g except that it maps $\downarrow_i$ to t. As Cresswell (1990) points out, this apparatus is expressively equivalent to explicit quantification over times, assuming time variables can only occur as arguments of 'At' and 'Present'. Consider the translation functions * and † defined as follows:

$$\text{Present}(t_i)^\dagger = \forall p(p \leftrightarrow \downarrow_i p) \qquad (\downarrow_i\varphi)^* = \text{At } t_i\varphi^*$$
$$(\text{At } t_i\ \varphi)^\dagger = \downarrow_i\varphi^\dagger \qquad (\uparrow_i\ \varphi)^* = \exists t_i(\text{Present}(t_i)) \wedge \varphi^*$$
$$(\forall t_i\varphi)^\dagger = \uparrow_j A\uparrow_i\downarrow_j\varphi^\dagger (\text{where } j \text{ is a new index})$$

(* and † commute with all other operators and quantifiers.) For any formula φ, $\varphi^\dagger$ contains no time variables and φ^* contains no time-storage or 'then' operators. It is easily shown that, whenever φ is a closed sentence in which each 'then' operator is in the scope of the corresponding storage operator, φ is logically equivalent to any formula obtained from φ by first relabeling its time variables so that they don't share any indices with any time-storage or 'then' operators and then applying either * or †. All of the above applies mutatis mutandis to quantification over worlds (see Correia 2007), for which one would want a separate set of "world-storage" operators $\Uparrow_i$ and "world-retrieval" operators $\Downarrow_i$.

[13] When $f: \mathrm{W} \to \mathrm{T}^{\mathrm{T}}$ maps each member of W to a permutation of T, define $f^*: \mathcal{P}(\mathrm{W}\times\mathrm{T}) \to \mathcal{P}(\mathrm{W}\times\mathrm{T})$ by $f^*(\mathrm{X}) = \{\langle w, f(w)(t)\rangle : \langle w, t\rangle \in \mathrm{X}\}$. A straightforward induction shows that for any $\mathcal{L}$-formula φ and assignment g, $[\![\varphi]\!]^{f^*\circ g} = f^*([\![\varphi]\!]^g)$. But this is not true for any of the listed expansions of $\mathcal{L}$. For example, consider the Montagovian model where $\mathrm{W} = \mathrm{T} = \{0,1\}$ and $\alpha{=}\eta{=}0$. For each $x \in \{0,1\}$, let $f(0)(x){=}x$ and $f(1)(x){=}1{-}x$. Let $g(p) = \{\langle 0,0\rangle, \langle 1,0\rangle\}$ so $f^*\circ g(p) = \{\langle 0,0\rangle, \langle 1,1\rangle\}$. Then $[\![\mathrm{N}p]\!]^g = \mathrm{W}\times\mathrm{T}$, so $[\![\Box\mathrm{N}p]\!]^g = \mathrm{W}\times\mathrm{T} = f^*(\mathrm{W}\times\mathrm{T}) = f^*([\![\Box\mathrm{N}p]\!])$, but $[\![\mathrm{N}p]\!]^{f^*\circ g} = \{\langle 0,0\rangle, \langle 0,1\rangle\}$, so $[\![\Box\mathrm{N}p]\!]^{f^*\circ g} = \phi \neq f^*([\![\Box\mathrm{N}p]\!])$. Similarly for $\exists t(\mathrm{Present}(t) \wedge \Box\mathrm{A}(\mathrm{Present}(t) \to p)$, $\exists t(\forall q(q \leftrightarrow \mathrm{At}\ tq) \wedge \Box\mathrm{At}\ tp)$, $\Box(p \leftrightarrow @p)$, and $\exists w(\forall q(q \leftrightarrow \mathrm{At}\,w\,q) \wedge \Box(p \leftrightarrow \mathrm{At}\,w\,q))$. By contrast, if we just add world-quantifiers and 'Actualized', with the obvious interpretations, every world-variable free formula is equivalent to an $\mathcal{L}$-formula, namely one derived from it by eliminating index-sharing, replacing Actualized(w_i) with p_i, and replacing $\forall w_i(\ldots)$ with $\forall p_i(\mathrm{WorldProp}^*(p_i) \to \ldots)$, where $\mathrm{WorldProp}^*(\varphi) =_{\mathrm{df}} \Diamond(\mathrm{A}\varphi \wedge \forall q(\mathrm{A}q \to \Box(\varphi \to q)))$.

[14] A product structure $\langle \mathrm{W}\times\mathrm{T}, =_2, =_1, \langle\alpha, \eta\rangle\rangle$ is in a natural sense the *product* of the pointed Kripke structures $\langle \mathrm{W}, \mathrm{U_W}, \alpha\rangle$ and $\langle \mathrm{T}, \mathrm{U_T}, \eta\rangle$, where $\mathrm{U_W}$ and $\mathrm{U_T}$ are the universal accessibility relations on W and T: see Kurucz 2007

[15] The interpretation function on generalized product structures can be naturally extended to interpret sentences containing 'Present', 'Actualized', '$\forall t$', and '$\forall w$' in the same way as in the case of product models. By contrast, there are several different possible ways of interpreting N, @, At t, and At w in such structures, for reasons that will emerge in §IX.

[16] *Proof*: we first show that in a relational structure where every point is square-completing, whenever y is reachable from x in three steps it is also reachable from x in two steps. Suppose $x \approx_\Box z_1$, $z_1 \approx_\mathrm{A} z_2$, and $z_2 \approx_\Box y$. Since z_2 is square-completing, there is some point z_3 such that $z_1 \approx_\Box z_3$ and $z_3 \approx_\mathrm{A} y$; then by the transitivity of $\approx_\Box$, $x \approx_\Box z_3$, so y is just two steps from x. (Similarly when the initial sequence contains two temporal and one modal step.) We can then show by induction that whenever y is reachable from x in any finite number of steps it is reachable in two steps. If the structure is connected, this means that any two points are just two steps apart — and given that the intermediate point is square-completing, the steps can be taken in either order.

[17] Notice that both *Symmetry* and *Church-Rosser* are symmetric with respect to modality and tense in the sense that they are equivalent (in the background logic) to their "mirror images", i.e. the results of interchanging $\Box$ and A in them. The mirror-image of *Symmetry*, $\forall p(\neg p \to \exists q(\mathrm{A}(p \to q) \wedge \Box(p \to \neg q)))$, is witnessed for a given p by the negation of any proposition q that witnesses *Symmetry* for that p. The mirror-image of *Church-Rosser*, $\forall p(\mathrm{S}\Box p \to \Box\mathrm{S}p)$, is equivalent to $\forall p(\neg\mathrm{A}\Diamond\neg p \to \neg\Diamond\mathrm{A}\neg p)$ and hence to $\forall p(\Diamond\mathrm{A}\neg p \to \mathrm{A}\Diamond\neg p)$, which is equivalent to *Church-Rosser* by quantificational logic.

[18] *Proof*: From $\Box$(*Church-Rosser*) we get $\forall p\Box(\Diamond\mathrm{A}p \to \mathrm{A}\Diamond p)$ (by the Converse Barcan Formula), hence $\forall p\Box(\Diamond\mathrm{A}\Box p \to \mathrm{A}\Diamond\Box p)$ by quantificational reasoning, hence $\forall p(\Box\Diamond\mathrm{A}\Box p \to \Box\mathrm{A}\Diamond\Box p)$ by the modal K schema, hence $\forall p(\mathrm{A}\Box p \to \Box\mathrm{A}p)$ by the modal B schema. Similarly, A(*Church-Rosser*) implies $\forall p(\Box\mathrm{A}p \to \mathrm{A}\Box p)$. The two together thus yield the commutativity principle $\forall p(\Box\mathrm{A}p \leftrightarrow \mathrm{A}\Box p)$, which by an induction using the temporal and modal 4 schemas implies $\forall p(\Box\mathrm{A}p \to \mathrm{O}_1 \ldots \mathrm{O}_n p)$ for any string $\mathrm{O}_1 \ldots \mathrm{O}_n$ of As and $\Box$s. In particular we have $\Box\mathrm{A}(\textit{Symmetry}) \to \mathrm{O}_1 \ldots \mathrm{O}_n(\textit{Symmetry})$. The combination of commutativity with A(*Church-Rosser*) also implies $\Box$A(*Church-Rosser*), and hence $\mathrm{O}_1 \ldots \mathrm{O}_n$(*Church-Rosser*) for any $\mathrm{O}_1 \ldots \mathrm{O}_n$. For consider any proposition p. If $\mathrm{A}\Diamond p$ is false then $\mathrm{A}\neg\mathrm{A}\Diamond p$, so $\mathrm{A}\neg\Diamond\mathrm{A}p$ by A(*Church-Rosser*), so $\mathrm{A}\Box\neg\Diamond\mathrm{A}p$, so $\Box\mathrm{A}\neg\Diamond\mathrm{A}p$ by commutativity, so $\Box\mathrm{A}(\Diamond\mathrm{A}p \to \mathrm{A}\Diamond p)$. If on the other hand $\mathrm{A}\Diamond p$ is true, we have $\mathrm{A}\Box\Diamond p$, so $\Box\mathrm{A}\Diamond p$ by commutativity, so $\Box\mathrm{A}\mathrm{A}\Diamond p$, so again $\Box\mathrm{A}(\Diamond\mathrm{A}p \to \mathrm{A}\Diamond p)$.

[19] Fritz (forthcoming) proves that the product logic (his Λ_{PU}) is not recursively axiomatizable; given its axiomatizability relative to the background logic, it follows that the background logic (Fritz's Λ_{PF}) is also not recursively axiomatizable, a fact already proved by Antonelli and Thomasson (2002).

[20] NOW is valid on the logic of product models extended to interpret the N operator in the manner described in §II, but not on the logic of Montagovian models extended in the same way. $\mathrm{RIG_N}$ is valid on both classes of models.

[21] NOW+ is also valid on the class of product models enriched to interpret quantification over times. If you are a fan of the approach to temporal anaphora discussed in footnote 12, then, for your eyes only, we can reformulate NOW+ as follows: $\Box\mathrm{A}{\uparrow_1}\forall p\Box(p \leftrightarrow {\downarrow_1}p)$.

[22] By contrast, there is no straightforward derivation of NOW+ from *Supervenience*. We can see this fact model-theoretically, by considering a generalization of product structures in which the modal accessibility relation need not be $=_2$ (sharing the same time co-ordinate), but can be any equivalence relation each of whose equivalence classes contains exactly one point with each world co-ordinate. *Symmetry* (and hence *Supervenience*) is still valid on this class of structures, but NOW and NOW+ no longer are.

[23] We also have doubts about (b) which will emerge in §IX.

[24] It would be a bit uncomfortable for us if, outside of special contexts like that of philosophy, modals in ordinary language always commuted with 'now': one might then worry that metaphysical necessity as we conceive it does not really count as a kind of necessity at all. But there are cases where ordinary modals seem not to commute with 'now'. Consider for example the use of 'might' on which it concerns the epistemic state of some salient person other than the speaker. If the salient person is not sure of the time, we can say things like 'Although she knows that it is raining, it might not (as far as she knows) be raining now, since she is still trying to find out what time it is'.

[25] Thanks to John Hawthorne for suggesting this argument.

[26] Note too that there is no need for defenders of *Perpetuity* to make a once-and-for-all choice as regards which premise to give up: given the context-sensitivity of counterfactuals, one could reasonably maintain that (i) holds in some contexts while (ii) holds in others.

[27] ACT and $\mathrm{RIG}_{@}$ are also both valid on the class of product models enriched to interpret @ in the manner described in §II.

[28] However, see Yalcin 2015 for reasons to doubt that $\mathrm{RIG}_{@}$ admits a true reading in ordinary English.

[29] There are a number of ways in which this multiplicity might be generated. On one view (Crossley and Humberstone 1977), the word 'actually' is ambiguous, having a 'rhetorical' use on which it is semantically inert, and a separate 'logical' use characterized by $\mathrm{RIG}_{@}$. On a second view (Correia 2007; inspired by Vlach 1973), 'actually' is more flexible, and has the effect that the sentence it embeds is evaluated as if it occurred at some wider scope in the sentence; on its most natural implementation, this view treats 'actually' as a bindable operator, introducing structural ambiguities analogous to those introduced by variables. On a third view (Yalcin 2015), 'actually' is strictly speaking semantically inert, although it may provide clues to the resolution of certain structural ambiguities which are present in the 'actually'-free sentence.

[30] If you are having trouble accessing the required non-epistemic reading of 'could' in (1a), consider instead 'He could work harder than he actually does'.

[31] Of course, (2b) has a trivially false reading too. But when the present tense occurs in a past-tense environment (with or without 'actually') a reading analogous to the true reading of (2b) is forced — e.g. 'It used to be colder than it [actually] is'. See Wehmeier 2004 and Mackay 2013.

[32] See Mackay 2017 for a presupposition-theoretic account of the felicity-conditions for 'actually'.

[33] The changing pluriverse picture is not the only way of combining propositional temporalism with the four principles. One could instead adopt a view on which facts about what is true at a given possible world are both permanent and necessary, but which things are possible worlds is a temporary matter, so that any given possible world is only possible for an instant.

[34] In the generalized Montagovian structures of §III, this condition is satisfied by all and only the singleton subsets of the domain.

[35] In generalized Montagovian structures this condition is satisfied by all and only the equivalence classes of temporally accessible points.

[36] Let p be some temporary truth. Then $p \wedge \neg \mathrm{A}p$ is true, and so possibly true, but not possibly always true, and hence not necessitated by any possible world-history, in violation of *Leibnizian Possibility*; and at least one possible history, viz. the true one, only necessitates truths and hence fails to necessitate the negation of $p \wedge \neg \mathrm{A}p$, in violation of *Negation*.

[37] Where p is a temporary truth, the true world-history is compossible both with p and with $\neg p$, in violation of *Negation*, and not compossible with $p \wedge \neg p$, in violation of *Conjunction*.

[38] These three further problematic features are not inevitable consequences of the historic conception of worlds. Instead of a necessitating or compossibility interpretation of 'At *w*', one could adopt a counterfactual interpretation, on which a proposition is true at a world if and only if it would have been

true had the corresponding world-history been true. Given the Strong Centering principle, according to which counterfactuals with true antecedents have the same truth value as their consequents, this analysis preserves the coextensiveness of truth with truth at the actual world. *Conjunction* holds; given *Perpetuity* and propositional temporalism, *Leibnizian Possibility* fails. In order to get *Disjunction* to hold we need Conditional Excluded Middle, in which case *Leibnizian Necessity* will also fail. *Negation* holds assuming that only counterfactuals with metaphysically impossible antecedents are vacuously true. Another interesting feature of the view is that, unlike the views considered in the main text, it preserves ACT. On the other hand $\mathrm{RIG}_{@}$ and its converse both fail (assuming *Perpetuity* and propositional temporalism), since the property of being true at the actual world is one that temporary truths have only contingently.

Another analysis yielding similar results holds that p is true at w just in case whichever time is present is such that possibly, w's world-history and p are both true and that time is present. But if the present time could fail to be ever present, as we will argue in §IX, this view will require giving up *Negation.*

[39] Bacon (2018) defends a view on which all *fundamental* truths are eternal, and on which they jointly necessitate, but do not eternally imply, the non-fundamental truths.

[40] This is not an inevitable commitment of propositional temporalism: Dorr (MS) develops a form of propositional temporalism on which only non-qualitative (haecceitistic) propositions can be temporarily true. The arbitrariness worries we are about to raise do not arise in any obvious way for proponents of *F-Supervenience* who accept this view.

[41] If being type-F is a contingent property of propositions, we could redefine an F-possibility as a possibly-true conjunction which, for each proposition, specifies whether it is type-F or not, and if it is specified to be type-F, specifies whether it is true or not, and also has *F-Supervenience* itself as a conjunct.

[42] Bacon (2018) suggests a response to this sort of worry: although s is specially related to a large family of properties and relations which includes *being a star, being spherical, being more massive than*, etc., this is just one of many structurally similar families of properties and relations, and every slice is specially related to its own such family. Bacon further claims that these families of properties and relations are all ‘on a par’, so that s is not special in any objectionable sense. But we think that in the relevant sense of ‘on a par’, namely being equally *natural* in the sense of Lewis (1983), the different families are *not* on a par. For the members of the familiar family that contains *being a star* are easier to refer to than their counterparts in other families (which typically require special devices like metric tense operators to express), which is good reason to think them more natural: see Dorr and Hawthorne 2012. (The fact that *reference* also has counterparts in other families, each of which is similarly easy for people to stand in to the members of that family, does not undermine this point.)

[43] (ii) will be denied by “modal anti-haecceitists”, according to whom all truths are necessitated by qualitative truths, and perhaps also, depending on the shape of spacetime, by “metric essentialists” like Maudlin (1990).

[44] For the expression of such claims see Burgess (2002). Relational structures can be straightforwardly extended to interpret G and H by supplementing them with a transitive, asymmetric relation $<$ on the domain, required to be such that $x \approx_{\mathrm{A}} y$ just in case $x = y$ or $x < y$ or $y < x$:

$$[\![\mathrm{G}\varphi]\!]^g = \{x \in \mathrm{I} : y \in [\![\varphi]\!]^g \quad \text{for all } y \text{ such that } x < y\}$$

$$[\![\mathrm{H}\varphi]\!]^g = \{x \in \mathrm{I} : y \in [\![\varphi]\!]^g \quad \text{for all } y \text{ such that } y < x\}$$

This language contains sentences which are true at a point if and only if the $<$-ordering restricted to points temporally accessible from it is discrete; similarly for being discrete and bounded, being unbounded, being dense, being dense and Dedekind-complete, and other properties which settle the cardinality of temporally accessible points.

[45] The tension between this way of thinking and *Symmetry* is perhaps unsurprising, since the idea that contingent questions can be “objective” in a way that temporary questions cannot be involves a breaking of the symmetry between time and modality.

[46] The reverse is true as well: *Eternity* follows in the background logic from the combination of $\Box$(*Church-Rosser*) and A(*Church-Rosser*), since as shown in §III these jointly imply the commutativity principle $\forall p\Box\mathrm{A}(\Box\mathrm{A}p \leftrightarrow \mathrm{A}\Box p)$ and hence $\forall p\Box\mathrm{A}(\Box\mathrm{AA}p \leftrightarrow \mathrm{A}\Box\mathrm{A}p)$, which is trivially equivalent to *Eternity*.

[47] In fact, just from the truth of *Church-Rosser* and A(*Symmetry*) in a model, it already follows that there is no point x modally accessible from ι such that x's history is smaller than ι's (where the history of a point is the set of all points temporally accessible from it). For by *Church-Rosser*, ι is square-completing, so for every point in ι's history, there is some point in x's history is modally accessible. If ι's history had a larger cardinality than x's, it would have to contain two distinct points from which the same point in x's history was modally accessible. But since modal accessibility and temporal accessibility are both equivalence relations, these two points would be both modally and temporally accessible to one another, and would therefore not be unaccompanied, which cannot happen if A(*Symmetry*) is true in the model. By similar reasoning, the truth of $\Box$(*Church-Rosser*) and $\Box$A(*Symmetry*) is enough to guarantee that no point modally accessible from ι has a history *greater* in cardinality than that of ι.

[48] Suppose, for example, that there are in fact at least three times, but there could have been at most two times: it is possible that although you only live twice, you live forever. That is:

(i) $\exists p_1\exists p_2\exists p_3(\mathrm{S}p_1 \wedge \mathrm{S}p_2 \wedge \mathrm{S}p_3 \wedge \neg\mathrm{S}(p_1 \wedge p_2) \wedge \neg\mathrm{S}(p_1 \wedge p_3) \wedge \neg\mathrm{S}(p_2 \wedge p_3))$

(ii) $\diamond\forall q_1\forall q_2\forall q_3((\mathrm{S}q_1 \wedge \mathrm{S}q_2 \wedge \mathrm{S}q_3) \to (\mathrm{S}(q_1 \wedge q_2) \vee \mathrm{S}(q_1 \wedge q_3) \vee \mathrm{S}(q_2 \wedge q_3))$.

We will derive a contradiction from (i), (ii), *Eternity*, and the permanent truth of *Symmetry*. Since *Symmetry* implies *Supervenience** (as explained in §III), *Supervenience** is always true: $\mathrm{A}\forall p(p \to \exists q(\mathrm{A}(q \to p) \wedge \Box q))$. Hence

$\forall p\mathrm{A}(p \to \exists q(\mathrm{A}(q \to p) \wedge \Box q))$ (by the temporal Converse Barcan Formula); so

$\forall p(\mathrm{S}p \to \mathrm{S}\exists q(\mathrm{A}(q \to p) \wedge \Box q))$ (by the normal modal logic of A); so

$\forall p(\mathrm{S}p \to \exists q\mathrm{S}(\mathrm{A}(q \to p) \wedge \Box q))$ (by the temporal Barcan Formula); so

$\forall p(\mathrm{S}p \to \exists q(\mathrm{A}(q \to p) \wedge \mathrm{S}\Box q))$ (by temporal S5).

Instantiating this generalization with the three propositions p_1, p_2, p_3 that exist according to (i) guarantees the existence of corresponding sometimes-necessary propositions q_1, q_2, q_3 which respectively always materially imply p_1, p_2, and p_3. By the modal 4 axiom, they are sometimes *necessarily* necessary: $\mathrm{S}\Box\Box q_1 \wedge \mathrm{S}\Box\Box q_2 \wedge \mathrm{S}\Box\Box q_3$. By the mirror-image Church-Rosser principle $\forall p(\mathrm{S}\Box p \to \Box\mathrm{S}p)$ (which follows from *Eternity* as explained above), $\Box\mathrm{S}\Box q_1 \wedge \Box\mathrm{S}\Box q_2 \wedge \Box\mathrm{S}\Box q_3$, hence $\Box(\mathrm{S}\Box q_1 \wedge \mathrm{S}\Box q_2 \wedge \mathrm{S}\Box q_3)$. So by (ii) (using the modal Converse Barcan Formula), $\Diamond(\mathrm{S}(\Box q_1 \wedge \Box q_2) \vee \mathrm{S}(\Box q_1 \wedge \Box q_3) \vee \mathrm{S}(\Box q_2 \wedge \Box q_3)$, and hence $\Diamond\mathrm{S}\Box(q_1 \wedge q_2) \vee \Diamond\mathrm{S}\Box(q_1 \wedge q_3) \vee \Diamond\mathrm{S}\Box(q_2 \wedge q_3)$. By $\Box$(*Church-Rosser*) (another consequence of *Eternity*), this implies $\Diamond\Box\mathrm{S}(q_1 \wedge q_2) \vee \Diamond\Box\mathrm{S}(q_1 \wedge q_3) \vee \Diamond\Box\mathrm{S}(q_2 \wedge q_3)$, and hence $\mathrm{S}(q_1 \wedge q_2) \vee \mathrm{S}(q_1 \wedge q_3) \vee \mathrm{S}(q_2 \wedge q_3)$. But since q_1, q_2, q_3 respectively always materially imply p_1, p_2 and p_3, this implies that $\mathrm{S}(p_1 \wedge p_2) \vee \mathrm{S}(p_1 \wedge p_3) \vee \mathrm{S}(p_2 \wedge p_3)$, contradicting (i). Unlike the argument about cardinality 1 in the main text, this reasoning generalizes straightforwardly to other finite cardinalities, but could be resisted by rejecting appeals to the temporal or modal Barcan or Converse Barcan formulae. However it unclear whether this way of resisting the argument leads to a stable view. Even if we say that none of q_1, q_2, and q_3 would have existed had there been only two times, it is hard to see how they could nevertheless all have been sometimes necessary without any two of them having been necessary together.

[49] Even someone who thought that the composition and topological structure of spacetime was metaphysically non-contingent might think that which regions of spacetime count as "simultaneity slices" is a contingent matter — e.g. if being a simultaneity slice is being a Cauchy surface of constant mean curvature (see Belot and Earman 2001: 239–40 and references therein) and there is a reasonable amount of contingency as regards the distribution of matter in spacetime.

[50] One deviant way of reconciling the above considerations with the letter of *Tomorrow Never Dies* is to adopt an eliminativist view on which there aren't any times at all. The less radical version of this view rejects the ontology of times but not the devices of temporal anaphora discussed in note 12. Assuming such devices are legitimate, we can use them to formulate eliminativism-friendly

analogues of *Tomorrow Never Dies*, *Die Another Day*, and *Live And Let Die*, to which the arguments of the present section are equally applicable. A more radical version of eliminativism holds that the entire notion of trans-history simultaneity is unintelligible, so that devices of temporal anaphora cannot meaningfully be embedded under modal operators. This picture renders the present section's arguments redundant, since its proponents already reject NOW (as unintelligible). Similar points apply to proposals to save the letter of *Tomorrow Never Dies* by holding that there is only one time, namely the present.

[51] Note that $\forall t \mathrm{A}(\mathrm{Present}(t) \rightarrow \Box\mathrm{Present}(t))$, and hence the disjunction of *Tomorrow Never Dies* and *Die Another Day*, is valid in the generalized product structures described in §III (interpreting 'Present' and '$\forall t$' as explained in §II), since in such models $\langle w, t\rangle$ is modally accessible from $\langle w', t'\rangle$ only when $t = t'$. These models provide the natural way of generalizing product models to allow for contingency in the composition or cardinality of the time series. Whereas *Tomorrow Never Dies* is valid in product models, it can fail in generalized product structures. For example, in a generalized product structure where $\mathrm{I} = \{\langle 0,1\rangle, \langle 0,2\rangle, \langle 1,0\rangle, \langle 1,2\rangle, \langle 2,0\rangle, \langle 2,1\rangle\}$, *Tomorrow Never Dies* is false at every point.

[52] (b), (c), and (d) are temporal analogues to the three accounts of truth at a world compatible with the combination of *Perpetuity* and *Historicity* considered in §VI. If it is contingent what times are sometimes present, (b) and (c) entail that 'At t' fails to commute with negation in modal contexts, whereas (d) preserves commutativity given Conditional Excluded Middle.

[53] Those who reject the factivity of 'always' have another way of rejecting the implication from accuracy to presence: they can say that for a time to be present is for it to be both accurate and sometimes accurate. We have already said what we have to say against this maneuver in connection with the step from (1) to (2).

[54] The views we have just considered all also entail that being present necessitates being accurate. For (a) this is obvious; for (b), (c), and (e) it follows from the principle that no time is present more than once (i.e. that if sometimes t is present and p is true, then always if t is present p is true); for (d) it follows from "strong centering" for counterfactuals, according to which a counterfactual with a true antecedent has the same truth value as its consequent. Relying only on the implication from presence to accuracy, we can run a different argument against NOW, replacing *Live And Let Die* with the stronger premise that the present time could have been absent from the time series without any new times being included in it (e.g. because time came to an end before now). We can articulate this possibility in a language with plural quantification over times, as follows:

> *The Living Daylights*: There are some *tt* such that none of *tt* is present; each of *tt* is sometimes present; and possibly always one of *tt* is present.

If such *tt* exist, then at least one of them t is possibly present, and so possibly accurate: $\Diamond\forall p(p \leftrightarrow \mathrm{At}\ t\, p)$. Instantiating NOW+ ($\Box\forall t$ At t $\forall p\Box(p \leftrightarrow \mathrm{At}\ t\, p)$) with t, we have $\Diamond\forall p\Box(p \leftrightarrow \mathrm{At}\ t\, p)$, hence $\Diamond\Box(\neg\mathrm{Present}(t) \leftrightarrow \mathrm{At}\ t\ \neg\mathrm{Present}(t))$, hence (by the modal B schema), $\neg\mathrm{Present}(t) \leftrightarrow \mathrm{At}\ t\ \neg\mathrm{Present}(t)$. Since t is not present this gives At t $\neg\mathrm{Present}(t)$. But, uncontroversially, every time is present at itself. So proponents of *The Living Daylights* must reject NOW+, and hence, as we argued in §V, should also reject NOW.

[55] Several people have suggested to us that proponents of *Symmetry* should retreat from NOW to the following weaker claim:

WEAK NOW Necessarily, if things are ever as they now are, things are as they now are.

$$\Box(\mathrm{S}\forall p(p \leftrightarrow \mathrm{N}p) \rightarrow \forall p(p \leftrightarrow \mathrm{N}p))$$

Advocates of *Symmetry* who are worried about NOW might take a quantum of solace in the fact that *Symmetry* can also be derived from WEAK NOW together with $\mathrm{RIG_N}$. (*Proof*: uncontroversially, $\forall q(q \leftrightarrow \mathrm{N}q)$. So $\mathrm{AS}\forall q(q \leftrightarrow \mathrm{N}q)$. Moreover, for any true p, we have $\mathrm{N}p$, hence by $\mathrm{RIG_N}$, $\mathrm{AN}p$. Thus $\mathrm{N}p \wedge \mathrm{S}\forall q(q \leftrightarrow \mathrm{N}q)$ is an eternal truth, and by WEAK NOW, one that necessitates p. So *Supervenience* is true: every truth is necessitated by an eternal truth. *Symmetry* follows.) But WEAK NOW seems completely unmotivated apart from being a consequence of NOW. Moreover, even WEAK NOW is in tension with certain natural claims about the extent of contingency in the composition of the time series, specifically *The Living Daylights* (see note 52). For just as proponents of NOW should accept the more general NOW+, proponents of WEAK NOW should accept the more general WEAK NOW+:

WEAK NOW+: Necessarily, at each time, it is necessary that if things are ever as they then are, things are as they then are.

$\Box\forall t \text{ At } t\, \Box(\mathrm{S}\forall p(p \leftrightarrow \text{At } t\, p) \rightarrow \forall p(p \leftrightarrow \text{At } t\, p))$

But WEAK NOW+ is inconsistent with *The Living Daylights* for the same reason that NOW+ is (see note 54): *The Living Daylights* entails that some temporarily non-present time t is possibly present, hence possibly accurate, hence by WEAK NOW+ possibly necessarily accurate-if-sometimes-accurate, hence in fact accurate, which no temporarily non-present time is.

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