Deep Structure

Jeremy Goodman (USC)

Logic and Metaphysics Workshop – March 11, 2019

1 Background logic

We'll work in a (simply, relationally typed) higher-order language: e is a type; every sequence of types is a type; nothing else is a type.

UI $\forall x^\tau \varphi \to \varphi[a^\tau/x]$

$\beta_\leftrightarrow$ $(\lambda x_1^{\tau_1} \dots x_n^{\tau_n}.\varphi)a_1^{\tau_1} \dots a_n^{\tau_n} \leftrightarrow \varphi[a_1/x_1; \dots ; a_n/x_n]$

$=_\tau := (\lambda x^\tau y^\tau . \forall Z^{\langle\tau\rangle}(Zx \leftrightarrow Zy))$

Observation 1. $\ulcorner Fa \urcorner = \ulcorner Gb \urcorner \Rightarrow F = G$

Proposition 2. $\forall p^{\langle\rangle} \neg\forall X^{\langle\langle\rangle\rangle} \forall Y^{\langle\langle\rangle\rangle}(Xp =_{\langle\rangle} Yp \to X =_{\langle\langle\rangle\rangle} Y)$.

2 Constituent Structure

Observation 3. $\varphi[a/x] = \psi[b/x] \not\Rightarrow \varphi = \psi$.

Observation 4. $\varphi[a/x] = \psi[b/x] \Rightarrow \varphi = \psi$, provided neither a nor b occurs in either φ or ψ.

read $x^\tau \leq_{\tau,\tau'} y^{\tau'}$ as "x is a *metaphysical constituent* of y"

$a_1^{\tau_1}, \dots, a_n^{\tau_n} \not\leq b_1^{\tau_1'}, \dots, b_m^{\tau_m'} := \bigwedge_{1\leq i\leq n; 1\leq j\leq m} \neg(a_i \leq_{\tau_i,\tau_j'} b_j)$

CONSTITUENT STRUCTURE
$\forall X \forall Y \forall x \forall y(x, y \not\leq X, Y \to (Xx =_{\langle\rangle} Yy \to X =_{\langle\tau\rangle} Y))$

worry: Being loved by John is a different property from loving Mary, and neither property is a metaphysical constituent either of being instantiated by John or of being instantiated by Mary.

$(\lambda z.Rxz), (\lambda z.Rzy) \not\leq (\lambda Z.Zx), (\lambda Z.Zy)$ and $(\lambda z.Rxz) \neq_{\langle e\rangle} (\lambda z.Rzy)$

So on any reasonable interpretation, CONSTITUENT STRUCTURE is incompatible with:

$\beta_=$ $(\lambda x_1^{\tau_1} \dots x_n^{\tau_n}.\varphi)a_1^{\tau_1} \dots a_n^{\tau_n} =_{\langle\rangle} \varphi[a_1/x_1; \dots ; a_n/x_n]$

3 Deep Structure

Idea: God's language has only a single sentence where our language has distinct sentences related by the substitution of subformulas of the form $\ulcorner(\lambda x_1 \dots x_n.\varphi)a_1 \dots a_n\urcorner$ and $\varphi[a_1/x_1; \dots ; a_n/x_n]$.

Observation 5. $\varphi[a/x]$ and $\psi[b/x]$ have the same normalization $\not\Rightarrow$ φ and ψ have the same normalization, even when neither a nor b occurs in either φ or ψ.

Observation 6. $\varphi[a/x]$ and $\psi[b/x]$ have the same normalization $\Rightarrow$ φ and ψ have the same normalization, provided neither a nor b occurs in either φ or ψ *and a and b are constants.*

DEEP STRUCTURE
$\forall X \forall Y \forall x \forall y((\mathcal{F}_\tau x \wedge \mathcal{F}_\tau y \wedge x, y \not\leq X, Y) \to (Xx =_{\langle\rangle} Yy \to X =_{\langle\tau\rangle} Y))$.

4 Deep Trouble

GENEALOGY
$\mathcal{F}_\tau x \to (x \not\leq y_1, \dots, y_n \to x \not\leq \Phi)$
where Φ contains no constants or free variables not among $y_1, \dots, y_n$.

ABSTRACTIVE STRUCTURE
$\forall p \forall q((\lambda z_1 \dots z_n.p) =_{\langle\tau_1,\dots,\tau_n\rangle} (\lambda z_1 \dots z_n.q) \to p =_{\langle\rangle} q)$.

SPECIFIABILITY
$\exists x \mathcal{F}_\tau^* x \to \exists G(\mathcal{I}(G) \wedge \exists x Gx \wedge \forall x(Gx \to \mathcal{F}_\tau^* x))$, where

- $\mathcal{I}$ ('is an independent specification') $:= (\lambda G.\forall x(Gx \to x \not\leq G))$
- Ł ('is sufficiently non-logical') $:= (\lambda x^\tau . x \not\leq \neg, \wedge, \forall_\tau, \forall_{\langle\tau\rangle}, \leq_{\tau,\langle\tau\rangle})$
- $\mathcal{F}_\tau^* := (\lambda x.\mathcal{F}_\tau x \wedge Łx)$

gloss: If there are any *sufficiently non-logical* fundamental entities, then there exists an *independent specification* of some of them.

Proposition 7. UI, $\beta_{\leftrightarrow}$, GENEALOGY, SPECIFIABILITY, ABSTRACTIVE STRUCTURE, and DEEP STRUCTURE imply that there are no sufficiently non-logical fundamental proposition, properties, or relations: $\neg\exists x \mathcal{F}^*_\tau x$ for $\tau \neq e$.

5 Why Specifiability?

SUFFICIENT HUMEANISM: $\forall x(\mathcal{F}_\tau x \to \text{Ł}x)$

+

ANTI-MELIANISM: $\mathcal{I}\mathcal{F}_\tau$

STRONG SPECIFIABILITY
$\forall x^\tau((\mathcal{F}_\tau x \wedge x \not\leq \neg, \wedge, \forall_\tau, \forall_{\langle\tau\rangle}, \leq_{\tau,\langle\tau\rangle}) \to x \not\leq \mathcal{F}_\tau)$

6 Whither structure?

6.1 Only individual constituents

FUNDAMENTAL INDIVIDUALS
$\forall x^e \mathcal{F}_e x$

INDIVIDUAL CONSTITUENCY
$\forall X^{\langle\tau_1,\ldots,\tau_n\rangle}\forall x^e(x \leq X \leftrightarrow \exists Y^{\langle e,\tau_1,\ldots,\tau_n\rangle}(X = (\lambda y_1 \ldots y_n.Rxy_1 \ldots y_n)))$

MEMORY
$\mathcal{F}_e x \to (x \not\leq \Phi \to x \not\leq y_1, \ldots, y_n)$
where Φ contains no constants or free variables not among $y_1, \ldots, y_n$

NON-VACUOUS $\beta_=$
$(\lambda x_1^{\tau_1} \ldots x_n^{\tau_n}.\varphi)a_1^{\tau_1} \ldots a_n^{\tau_n} =_{\langle\rangle} \varphi[a_1/x_1; \ldots; a_n/x_n]$
provided $x_1, \ldots, x_n$ all occur free in φ.

6.2 Only non-propositional constitutents

Give up NON-VACUOUS $\beta_=$ and ABSTRACTIVE STRUCTURE to have

NON-PROPOSITIONAL ATOMIC STRUCTURE[1]
$\forall X \forall Y \forall x_1 \ldots x_n \forall y_1 \ldots y_n(Xx_1 \ldots x_n =_{\langle\rangle} Yy_1 \ldots y_n \to X =_\tau Y)$, where τ is a non-propositional type

[1] e is a non-propositional type; every non-empty sequence of non-propositional types is a non-propositional type; nothing else is a non-propositional type.

Proof of Proposition 7

Assume for reductio that the antecedent of SPECIFIABILITY is satisfied for $\tau = \langle\tau_1, \ldots, \tau_n\rangle$, and consider some G of type $\langle\langle\tau_1, \ldots, \tau_n\rangle\rangle$ that witnesses the truth its consequent, so $\mathcal{I}(G) \wedge \exists x Gx \wedge \forall x(Gx \to \mathcal{F}^*_\tau x)$. We introduce the following definitions and abbreviations, omitting type indications wherever possible:

- let $\to$, $\leftrightarrow$, $\exists_\tau$ and $=_\tau$ be defined in terms of $\neg$, $\wedge$, $\forall_\tau$, and $\forall_{\langle\tau\rangle}$
- let $\overline{z}$ abbreviate $z_1^{\tau_1} \ldots z_n^{\tau_n}$
- $X!x := \forall z(Gz \to z \not\leq X) \wedge Gx$
- $B := (\lambda x.\exists Y \exists y(Y!y \wedge x = (\lambda\overline{z}.Yy) \wedge \neg Yx))$

The only constants occurring in B are $G, \neg, \wedge, \forall_\tau, \forall_{\langle\tau\rangle}$, and $\leq_{\tau,\langle\tau\rangle}$. And $\forall x(Gx \to x \not\leq G, \neg, \wedge, \forall_\tau, \forall_{\langle\tau\rangle}, \leq_{\tau,\langle\tau\rangle})$, since G is an independent specification of sufficiently non-logical entities. So GENEALOGY implies that $\forall x(Gx \to x \not\leq B)$, and hence $\forall x(Gx \to B!x)$. Since $\exists x Gx$, it follows that $\exists x(B!x)$. Our reductio will proceed by deriving $\neg B!a$ for arbitrary a.

Observe that $X!x \wedge Y!y$ implies $x, y \not\leq X, Y$ (by the definition of !) and $\mathcal{F}_\tau x \wedge \mathcal{F}_\tau y$ (by the characterization of G). DEEP STRUCTURE then yields the principle $(*) : \forall X \forall Y \forall x \forall y(X!x \wedge Y!y \wedge Xx = Yy \to X = Y)$. We will now derive $\neg B!a$ using only $(*)$, ABSTRACTIVE STRUCTURE, UI, $\beta_\leftrightarrow$, and classical quantificational reasoning.

1. $B(\lambda\overline{z}.Ba) \to \exists Y \exists y(Y!y \wedge (\lambda\overline{z}.Ba) = (\lambda\overline{z}.Yy) \wedge \neg Y(\lambda\overline{z}.Ba))$ [$\beta_\leftrightarrow$]
2. $B(\lambda\overline{z}.Ba) \to \exists Y \exists y(Y!y \wedge (\lambda\overline{z}.Ba) = (\lambda\overline{z}.Yy) \wedge B \neq Y)$ [1, Leibniz's Law]
3. $\forall Y \forall y(B!a \wedge Y!y \wedge Ba = Yy \to B = Y)$ [$(*)$, UI]
4. $\forall Y \forall y((\lambda\overline{z}.Ba) = (\lambda\overline{z}.Yy) \to Ba = Yy)$ [ABSTRACTIVE STRUCTURE, UI]
5. $B(\lambda\overline{z}.Ba) \to \neg B!a$ [2,3,4]
6. $\exists Y \exists y(Y!y \wedge (\lambda\overline{z}.Ba) = (\lambda\overline{z}.Yy) \wedge \neg Y(\lambda\overline{z}.Ba)) \to B(\lambda\overline{z}.Ba)$ [$\beta_\leftrightarrow$]
7. $B!a \wedge (\lambda\overline{z}.Ba) = (\lambda\overline{z}.Ba) \wedge \neg B(\lambda\overline{z}.Ba) \to$
   $\exists Y \exists y(Y!y \wedge (\lambda\overline{z}.Ba) = (\lambda\overline{z}.Yy) \wedge \neg Y(\lambda\overline{z}.Ba))$ [UI]
8. $\neg B!a$ [5,6,7]