Atomic structure and its limits

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Definition 1 (Relational types)**.** The set T of *relational types* is the smallest set containing e and t that is closed under the formation of finite sequences; we assume that $e \neq t \neq \langle\rangle$. Throughout τ will range over T.

Definition 2 (Non-propositional and ancestrally propositional types)**.** The set N of *non-propositional types* is the smallest set containing e that is closed under the formation of finite sequences. The set A of *ancestrally propositional types* $= T \backslash N$. Throughout ν will range over N and α will range over A.

Definition 3 (Terms)**.** We have infinitely many variables of every type. Additionally, $\ulcorner X(x_1 \dots x_n) \urcorner$ is a term of type t iff, for some type $\langle \tau_1, \dots, \tau_n \rangle$, X is a term of that type and the x_i are respectively of types τ_i; $\ulcorner (\lambda x_1 \dots x_n . X) \urcorner$ is a term of type $\langle \tau_1, \dots, \tau_n \rangle$ iff X is a term of type t and the x_i are pairwise-distinct variables respectively of types τ_i. Note that, if X is a term of type t, then $\ulcorner (\lambda . X) \urcorner$ is a term of type $\langle\rangle$ and not a term of type t, and if Y is a term of type $\langle\rangle$ then $\ulcorner Y() \urcorner$ is a term of type t and not a term of type $\langle\rangle$.

Convention 4 (Sequence notation)**.** When clear from context, we let $\overline{x}$ abbreviate $\langle x_1, \dots, x_n \rangle$; $\Pi \overline{x}$ abbreviate $x_1 \times \dots \times x_n$; $\langle x_1 \dots x_n \rangle + \langle y_1, \dots, y_m \rangle$ abbreviate $\langle x_1 \dots x_n, y_1, \dots, y_m \rangle$; $\Sigma \overline{x}$ abbreviate $x_1 + \dots + x_n$; $\pi_i(\langle x_1, \dots, x_n \rangle)$ abbreviate x_i; $f\overline{x}$ abbreviate $f(\overline{x})$; for convenience we equate singleton sequences with their members and functions f with domain $\{\langle\rangle\}$ with $f(\langle\rangle)$.

Definition 5 (Background logic)**.** We assume classical propositional logic and quantification theory, including unrestricted universal instantiation $\forall x \varphi \to \varphi[a/x]$, and that β-equivalent formulas are materially equivalent: $(\lambda x_1 \dots x_n . \varphi) \leftrightarrow \varphi[a_i/x_i]$. Higher-type identity predicates are defined in terms of indiscernibility.

Theorem 6 (Generalized Russell-Myhill)**.** $\forall x^\alpha \neg \forall X \forall Y (Xx = Yx \to X = Y)$

I'm going to be exploring views that accept, in some sense, as much atomic structure as possible; namely the schemas:

N-ATOMIC STRUCTURE
$X^\nu(x_1 \dots x_n) = Y(y_1 \dots y_n) \to X = Y \wedge \bigwedge_i x_i = y_i$

N-TYPOLOGY
$X^\nu(x_1 \dots x_n) \neq Y^{\nu'}(y_1 \dots y_m)$ for $\nu \neq \nu'$

Identifications of predications at a given non-propositional type imply the identifications of their respective ingredients, and no predication at a given non-propositional type is identical to any predication at any other non-propositional type. Either one of these principles is incompatible with identity-strength β-conversion. Notably, N-TYPOLOGY is incompatible with

FLATTENING
$p = (\lambda.p)()$

which is the nullary case of β-conversion. Intuitively, whatever the structure of p, the structure of $(\lambda.p)()$ is that of a nullary predication, since $\langle\langle\rangle\rangle \in N$. By contrast, all the views I'll be considering uphold

η-CONVERSION
$X = (\lambda x_1 \ldots x_n.X(x_1 \ldots x_n))$

including the nullary case $X = (\lambda.X())$.

In general, atomic structure requires predicate abstraction to sometimes forget structure in the formulas being abstracted on: features of the embedded formula that would make a difference for propositional identity when the formula occurs unembedded cannot always make for differences in property expressed by the resulting predicate abstract. This is because:

Theorem 7 (Another Russell-Myhill Generalization)**.**
$\forall X \forall Y (Xx^{\overline{\tau}} = Yx \rightarrow X = Y) \rightarrow \neg \forall p \forall q ((\lambda \overline{y^{\tau}}.p) = (\lambda \overline{y}.q) \rightarrow p = q)$

There is a general moral here. N-ATOMIC STRUCTURE and N-TYPOLOGY are claims about the structure of *predications* – i.e. (certain) propositions/entities of type t – and they imply surprisingly little about the granularity of *predicables*.

Indeed, in what follows I will be exploring views which validate:

BOOLEANISM
$(\lambda x_1 \ldots x_n.\varphi) = (\lambda x_1 \ldots x_n.\psi)$ for $\varphi \leftrightarrow \psi$ a classical tautology

This package takes some getting used to. For example, BOOLEANISM and η-CONVERSION imply that conjunction is a *symmetric relation* between nullary predicables, while N-ATOMIC STRUCTURE implies that conjoining nullary predicables is not a *commutative operation*:

$\&^{\langle\langle\rangle,\langle\rangle\rangle} := (\lambda X^{\langle\rangle} Y^{\langle\rangle}.X() \wedge Y())$

$\& = (\lambda XY.\&(YX))$ [by BOOLEANISM and η-CONVERSION]

$\&((\lambda.p)(\lambda.\neg p)) \neq \&((\lambda.\neg p)(\lambda.p))$ [by N-ATOMIC STRUCTURE]

In addition to having a Boolean level of granularity, the models below also imply that the space of non-propositional predicables is especially well-behaved, in the sense that the space of $n+1$-adic relations of type $\langle \nu_0, \ldots, \nu_n \rangle$ is isomorphic to the space of functions from entities of type ν_0 to n-adic relations of type $\langle \nu_1, \ldots, \nu_n \rangle$:

N-FUNCTIONALISM
$\forall x^{\nu_0} \exists! F^{\langle \nu_1, \ldots, \nu_n \rangle} R(x, F) \rightarrow \exists! G^{\langle \nu_0, \ldots, \nu_n \rangle} \forall x R(x, (\lambda y_1 \ldots y_n.Gxy_1 \ldots y_n))$

1 Strong typology

N-TYPOLOGY says that, among the non-propositional types, predications can be thought of as having a distinguished type. The following schema generalizes this situation to all types:

> STRONG TYPOLOGY
> $X^{\tau}(x_1 \dots x_n) \neq Y^{\tau'}(y_1 \dots y_m)$ for $\tau \neq \tau'$

Similarly, we might consider generalizing N-FUNCTIONALISM to all types thus:

> STRONG FUNCTIONALISM
> $\forall x^{\tau_0} \exists! F^{\langle \tau_1, \dots, \tau_n \rangle} R(x, F) \to \exists! G^{\langle \tau_0, \dots, \tau_n \rangle} \forall x R(x, (\lambda y_1 \dots y_n . G x y_1 \dots y_n))$

I will now describe a class of models of the resulting theory. Intuitively, there are two kinds of propositions: atomic predications of some non-propositional type, sets of worlds tagged with some ancestrally propositional type.

Definition 8 (Domains)**.**

$D_e :=$ some set

$W :=$ some non-empty set

$D_t := (\mathcal{P}(W) \times A) \cup \bigcup_{\overline{\tau} \in N} \mathcal{P}(W)^{\Pi \overline{D_\tau}} \times \Pi \overline{D_\tau}$

$D_{\overline{\nu}} := \{ f \in D_t^{\Pi \overline{D_\nu}} : \exists x \in \mathcal{P}(W)^{\Pi \overline{D_\nu}}, \forall \overline{y} \in \Pi \overline{D_\nu}, f(\overline{y}) = \langle x, \overline{y} \rangle \}$

$D_{\overline{\tau}} := (\mathcal{P}(W) \times \{\tau\})^{\Pi \overline{D_\tau}}$ for $\overline{\tau} \in A$

Definition 9 (Application)**.** $[\![F(\overline{a})]\!]^{\sigma} := [\![F]\!]^{\sigma}(\overline{[\![a]\!]^{\sigma}})$

Definition 10 (Truth conditions)**.** $\mathrm{tc} \in \mathcal{P}(W)^{D_t} : \langle p, \alpha \rangle \mapsto p$; $\langle x, \overline{y} \rangle \mapsto x(\overline{y})$ for $\overline{y} \notin \tau$

Definition 11 (Abstraction)**.**

$[\![(\lambda \overline{x^{\nu}}.\varphi)]\!]^{\sigma}(\overline{y}) := \langle x, \overline{y} \rangle$ where $x \in \mathcal{P}(W)^{\Pi \overline{D_\nu}} : \overline{z} \mapsto \mathrm{tc}([\![\varphi]\!]^{\sigma[x_i \mapsto z_i]})$

$[\![(\lambda \overline{x^{\tau}}.\varphi)]\!]^{\sigma}(\overline{y}) := \langle \mathrm{tc}([\![\varphi]\!]^{\sigma[x_i \mapsto y_i]}), \overline{\tau} \rangle$ for $\overline{\tau} \in A$

Definition 12 (Logical constants)**.**

$[\![\neg]\!]^{\sigma}(p) := \langle W \backslash \mathrm{tc}(p), \langle t \rangle \rangle$

$[\![\wedge]\!]^{\sigma}(\langle p, q \rangle) := \langle \mathrm{tc}(p) \cap \mathrm{tc}(q), \langle t, t \rangle \rangle$

$[\![\forall_\alpha]\!]^{\sigma}(f) := \langle \bigcap_{x \in D_\alpha} \mathrm{tc}(f(x)), \langle \langle \alpha \rangle \rangle \rangle$

$[\![\forall_\nu]\!]^{\sigma}(f) := \langle u, f \rangle$ where $u(g) := \bigcap_{x \in D_\tau} \mathrm{tc}(g(x))$

2 Adverbialism

How well motivated is STRONG TYPOLOGY in a setting where we only have atomic structure for non-propositional types? Not very, I think. The distinction between atomic propositions and type-tagged sets of worlds is inelegant too. Once we reject atomic structure at ancestrally propositional types, it would be nice to have a more elegant picture of the granularity of predications of entities of such types. One such conception begins with the paradigms

$$(\lambda p.p)\varphi = \varphi$$

$$\neg F\overline{a} = (\lambda \overline{x}.\neg Fx)\overline{a}$$

$$F\overline{a} \wedge G\overline{b} = (\lambda \overline{x}\overline{y}.F\overline{x} \wedge G\overline{y})\overline{a}\overline{b}$$

Here is the general idea. N-ATOMIC STRUCTURE and N-TYPOLOGY say that, when we predicate something of some entities all of non-propositional type, in a given order, the result is a predication with those entities as all and only its arguments, in that order. We remember the arguments to such predications. But now suppose the relation being predicated also takes some propositions as arguments. Why suddenly forget everything about the arguments you put in? Instead, we can remember the non-propositional arguments, and also keep track of the arguments of the propositions that we're predicating the relation of. What if the relation also takes as arguments entities of some ancestrally propositional type $\overline{\tau} \in A$? Well, we know from Russell-Myhill that we can't have the resulting propositions remember which such things we put in, since that would amount to injecting entities of that type into the space of propositions, in violation of Cantor's theorem. So the simplest thing to say is that they leave no trace in the argument structure of the resulting proposition. Picturesquely, think of propositions as atoms, the predicative components of which are the nuclei, with non-propositional entities as electrons, and ancestrally propositional relations as force fields that induce a kind of nuclear fusion, the result of which has all the electrons from the fusion materials around a new nucleus. The picture fits with the idea of sentential operators as being like adverbial predicate modifiers. It's also reminiscent of adverbialism in the philosophy of perception: the structure of the proposition that o appears F to me is that the relation of-being-appeared-to-F-ly relates me to o. Appearing, a propositional attitude of type $\langle e, t\rangle$, takes me and a predication of F to o and produces a predication of me and o.

Here is a strategy for making this picture more precise.

Definition 13 (Explicative abstraction)**.**

$\ulcorner(\lambda x_1 \ldots x_n.\varphi)\urcorner$ is *in good order* := each of $x_1, \ldots, x_n$ occurs exactly once in φ, and in that order.

α is a *naked argument* in β := α has an occurrence in β that is in argument position and that is not in the scope of any λ.

$\ulcorner(\lambda x_1 \ldots x_n.\varphi)\urcorner$ is *explicative* := it is in good order, $x_1, \ldots, x_n$ are each naked arguments in φ, and every naked argument in φ contains at least one of $x_1, \ldots, x_n$.

The above three principles about truth, negation, and conjunction can then be motivated as instances of the more general schema:

EXPLICATIVE-β
$(\lambda\overline{x}.\varphi)\overline{a} = \varphi[a_i/x_i]$ for explicative $(\lambda\overline{x}.\varphi)$

I'll now describe a class of structures that – I think! – validate this principle together with the principles advertised in the introduction.

Definition 14 (Argumentative content)**.**
$\mathrm{arg}(\langle x, \overline{y}\rangle) = \overline{y}$ for $\langle x, \overline{y}\rangle \in D_t$; $\mathrm{arg}(x) := \langle x\rangle$ for $x \in D_\nu$; $\mathrm{arg}(x) = \langle\rangle$ otherwise

Definition 15 (Adverbial domains)**.**
$D_t := \bigcup_{\overline{\tau}\in N} \mathcal{P}(W)^{\Pi\overline{D_\tau}} \times \Pi\overline{D_\tau}$
$D_{\overline{\nu}} := \{f \in D_t^{\Pi\overline{D_\nu}} : \exists x \in \mathcal{P}(W)^{\Pi\overline{D_\nu}}, \forall\overline{y} \in \Pi\overline{D_\nu}, f(\overline{y}) = \langle x, \overline{y}\rangle\}$ (as above)
$D_{\overline{\tau}} := \{f \in D_t^{\Pi\overline{D_\tau}} : \pi_2(f(\overline{y})) = \Sigma\overline{\mathrm{arg}(y)}$ for all $\overline{y} \in \Pi\overline{D_\tau}\}$ for $\overline{\tau} \in A$

Definition 16 (Relational correlate)**.** For $x \in \mathcal{P}(W)^{\Pi\overline{D_\nu}}$, let $\mathrm{rel}(x) :=$ the unique $f \in D_{\overline{\nu}}$ such that, for all $\overline{y} \in \Pi\overline{D_\nu}$, $\pi_1(f(\overline{y})) = x$.

Definition 17 (Logical form)**.** For $\langle x, \overline{y}\rangle \in D_t$, let $\mathrm{lf}(\langle x, \overline{y}\rangle) := \mathrm{rel}(x) + \overline{y}$.

Definition 18. $@(\langle X^{\langle\tau_1,\ldots,\tau_n\rangle}, \langle x^{\tau_1}, \ldots, x^{\tau_n}\rangle\rangle) := \ulcorner X(x_1 \ldots x_n)\urcorner$

Definition 19 (Explication)**.** A term $\ulcorner(\lambda\overline{y}.\psi)\urcorner$ *fully explicates* a term $\ulcorner(\lambda\overline{x}.\varphi)\urcorner$ *relative to* an assignment $\sigma :=$ for some function f from variables to sequences of variables:

(i) $\sigma(f(x_i)) = \mathrm{lf}(\sigma(x_i))$ for x_i of type t;

(ii) $f(x_i) = \langle x_i\rangle$ for x_i of type $\nu \in N$;

(iii) $f(x_i) = \langle\rangle$ for x_i of type $\overline{\tau} \in A$;

(iv) $\overline{y} = \Sigma\overline{f(x)}$;

(v) no member of $f(x_i)$ is free in φ;

(vi) $@(f(x_i))$ is free for x_i in φ for all x_i of type t;

(vii) $\psi = \varphi[@(f(x_i))/x_i : x_i$ of type $t]$.

A term $\ulcorner(\lambda\overline{y}.\psi)\urcorner$ *argumentatively explicates* a term $\ulcorner(\lambda\overline{x}.\varphi)\urcorner$ *relative to* an assignment $\sigma :=$ for some functions f, f^- from variables to sequences of variables:

(i′) f satisfies conditions (i)-(iii), (v)-(vii) of the definition of full explication

(ii′) $f(x_i) = \pi_1(f(x_i)) + f^-(x_i)$ if x_i is of type t; otherwise, $f(x_i) = f^-(x_i)$

(iii′) $\overline{y} = \Sigma\overline{f^-(x)}$;

(iv′) $\Sigma\overline{f(x)}$ is repetition-free

Proposition 20. For any term $\ulcorner(\lambda\overline{x^\tau}.\varphi)\urcorner$ and assignment σ there is a term $\ulcorner(\lambda\overline{y^\tau}.\psi)\urcorner$ that fully explicates the former relative to the latter. Any two terms that fully explicate a given term relative to a given assignment are α-equivalent; any such term will be identical to the term being explicated if that term is of non-propositional type; the same is true for argumentative explication.

Definition 21 (Adverbial abstraction)**.**

$[\![(\lambda\overline{x^\nu}.\varphi)]\!]^\sigma(\overline{y}) := \langle x, \overline{y}\rangle$ where $x \in \mathcal{P}(W)^{\Pi\overline{D_\nu}} : \overline{z} \mapsto \mathrm{tc}([\![\varphi]\!]^{\sigma[x_i \mapsto z_i]})$ (as above)

$[\![(\lambda\overline{x^\tau}.\varphi)]\!]^\sigma(\overline{\sigma(x)}) := [\![(\lambda\overline{y^\nu}.\psi)]\!]^\sigma(\overline{\sigma(y)})$

where $\ulcorner(\lambda\overline{y}.\psi)\urcorner$ argumentatively explicates $\ulcorner(\lambda\overline{x}.\varphi)\urcorner$ relative to σ

Remark 22. The above proposition together with the fact that the interpretation of a term relative to an assignment is insensitive to the value of the assignment for variables not free in the term implies that the above definition succeeds in specifying the interpretation of all λ-terms.

Example 23. We will show that $[\![(\lambda p^t.p)]\!]^\sigma(\langle x, \overline{y}\rangle) = \langle x, \overline{y}\rangle$ for some (and hence for every) assignment σ. Where $\overline{\nu}$ is the type of $\mathrm{rel}(x)$, let $Z^{\overline{\nu}}$ and $z_i^{\nu_i}$ be some pairwise-distinct variables and let σ be some assignment such that $\sigma(p) = \langle x, \overline{y}\rangle, \sigma(Z) = \mathrm{rel}(x)$, and $\sigma(z_i) = y_i$. The existence of functions f and f^- such that $f(p) = Z + \overline{z}$ and $f^-(p) = \overline{z}$ implies that $\ulcorner(\lambda Z\overline{z}.Z\overline{z})\urcorner$ and $\ulcorner(\lambda\overline{z}.Z\overline{z})\urcorner$ respectively fully and argumentatively explicate $\ulcorner(\lambda p.p)\urcorner$ relative to σ. So $[\![(\lambda p.p)]\!]^\sigma(\langle x, \overline{y}\rangle) = [\![(\lambda\overline{z}.Z(\overline{z}))]\!]^\sigma(\overline{y})$. By the clause for application, $\mathrm{tc}([\![Z(\overline{z})]\!]^{\sigma[z_i \mapsto w_i]}) = \mathrm{tc}(\mathrm{rel}(x)(\overline{w})) = x(\overline{w})$ for any $\overline{w} \in \Pi\overline{D_\nu}$. So $[\![(\lambda\overline{z}.Z(\overline{z}))]\!]^\sigma(\overline{y}) = \langle x, \overline{y}\rangle$.

Definition 24 (Logical constants)**.**

$[\![\neg]\!]^\sigma(\langle x, \overline{y}\rangle) := \langle {\sim}x, \overline{y}\rangle$

where ${\sim}x(\overline{z}) := W \backslash x(\overline{z})$ for all $\overline{z} \in \mathrm{dom}(x)$

$[\![\wedge]\!]^\sigma(\langle\langle x, \overline{y}\rangle, \langle x^*, \overline{y^*}\rangle\rangle) := \langle x \& x^*, \overline{y} + \overline{y^*}\rangle$

where $x \& x^*(\overline{z} + \overline{z^*}) := x(\overline{z}) \cap x^*(\overline{z^*})$ for all $\overline{z} \in \mathrm{dom}(x), \overline{z^*} \in \mathrm{dom}(x^*)$

$[\![\forall_\alpha]\!]^\sigma(f) := \langle \bigcap_{x \in D_\alpha} \mathrm{tc}(f(x)), \langle\rangle\rangle$

$[\![\forall_\nu]\!]^\sigma(f) := \langle u_\nu, f\rangle$

where $u_\nu(g) := \bigcap_{x \in D_\tau} \mathrm{tc}(g(x))$

Proposition 25 (Adverbial granularity)**.** For Boolean connectives, we get involution, de Morgan, and that propositions form a monoid w.r.t. conjunction.

Remark 26 (Saying stuff in the object language)**.** We can define in the object language what it is for an entity x^ν to be a constituent in argument position of the proposition p^t as follows: $\exists q^t \exists F^{\langle\nu\rangle} \exists r^t (p = (q \wedge Fx \wedge r))$. We can also simulate type-neutral quantification over all entities of all non-propositional types at once, as follows: say that p is a *tautology* $:= (\lambda.p) = (\lambda.p \to p)$; p is *nullary* $:= p = (\lambda.p)()$; p has *one argument* $:= \forall q \forall r (p = (q \wedge r) \to (\mathrm{nullary}(q) \leftrightarrow$

$\neg$nullary$(r)))$; p is a *proxy for some non-propositional entity* $:= p$ is a tautology with one argument. Constituency can be defined for these proxies as above, so we can say what it is for two propositions to have the same arguments. We can then define what it is for proxies to be proxies of entities of type e and of type $\langle\rangle$, and then draw type distinctions among other proxies by exploiting N-FUNCTIONALISM. This allows us to articulate, by way of these proxies, the theory of adverbial N-atomic structure in the form of object-language generalizations, rather than merely model-theoretically or schematically.

3 The multiple relation theory

It might seem an arbitrary limitation of adverbialism that, when we predicate X of some some proposition p, the *arguments* of p are 'remembered' as arguments of Xp, but the *relational constituent* of p is forgotten. Why not remember it too? From a technical perspective, this can be easy achieved as follows:

Definition 27 (Constituents)**.**

For $x \in D_\tau$, $\mathrm{con}(x) := \mathrm{lf}(x)$ if $\tau = t$ and $= \mathrm{arg}(x)$ if $\tau \neq t$

Definition 28 (Multiple relation models)**.** We modify the adverbial models by (i) replacing "arg" with "con" in the definition of $D_{\overline{\tau}}$, and (ii) replacing "argumentatively explicates" with "fully explicates" in the definition of $[\![(\lambda\overline{x^\tau}.\varphi)]\!]^\sigma$.

Despite this relatively minor difference in definitions, the resulting theory has quite different behavior. It is, in a way, reminiscent of Russell's so-called multiple-relation theory of judgment. On that view, when we predicate the relation of belief$^{\langle e,t\rangle}$ of John^e and the proposition that Mary^e sings$^{\langle e\rangle}$, the resulting proposition predicates a relation of John, Mary, and singing, which we might call doxastic-predication$^{\langle e,\langle e\rangle,e\rangle}$. If we were to say that Sam believes that John believes that Mary sings, the result would be the holding of a different relation – doxastic-predication$^{\langle e,\langle e,\langle e\rangle,e\rangle,e,\langle e\rangle,e\rangle}$ – of Sam, doxastic-predication$^{\langle e,\langle e\rangle,e\rangle}$, John, singing, and Mary. By contrast, adverbialism would have the resulting predication be the holding of a three-place relation between Sam, John, and Mary.

The resulting theory loses EXPLICATIVE-β, involution, de Morgan, and the associativity of conjunction. Indeed, conjunction becomes anti-idempotent: no proposition the result of conjoining it with itself. Moreover, all operators are anti-idempotent: $\forall X^{\langle t\rangle}\forall p(Xp \neq p)$. For example, every time we predicate truth of a proposition, we take another step along Bradley's regress: $(\lambda p.p)(X^{\overline{\nu}}(\overline{x^\nu})) = (\lambda Y\overline{y}.Y(\overline{y}))(X\overline{x^\nu})$. Similarly, existential generalizations of non-propositional type have higher-type predicative components than their instances. The construction thereby suggests the following solution to the sorts of puzzles about grounding that arise when we take truths to ground facts about their truth, true instances of existential generalizations to ground those generalizations, and grounding to be asymmetric: restrict the generalization principle to types at which propositions have the relevant sort of structure. Here is an example of the response in action.

Example 29. $[\![\exists_t(\lambda p.p)]\!] = \langle W, \langle\rangle\rangle := \top$. $[\![(\lambda p.p)(\exists_t(\lambda p.p))]\!] = \langle I_{\langle\rangle}, \top\rangle$, where $I_{\overline{\nu}}$ is the member of $\mathcal{P}(W)^{\overline{\Pi D_\nu}}$ such that $\mathrm{rel}(I_{\overline{\nu}}) = [\![(\lambda Y^{\overline{\nu}}\overline{y}.Y(\overline{y}))]\!]$. This proposition is a true instance of the non-propositional generalization $\exists_{\langle\rangle}((\lambda X^{\langle\rangle}.I_{\langle\rangle}(X)))$, and so we may suppose a ground of it. But $I_{\langle\rangle}$ – nullary instantiation, a property of nullary relations – should not be confused with $(\lambda p.p)$ – a property of propositions. So there is no challenge here to, for example, the grounding of the truth of any true proposition in that proposition and the grounding of true existential generalizations in their instances (for non-propositional-type generalizations).