An argument for necessitism*

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This paper presents a new argument for *necessitism*, the claim that necessarily everything is necessarily something. The argument appeals to principles about the metaphysics of quantification and predication which are best seen as constraints on reality's fineness of grain. I give this argument in section 4; the impatient reader may skip directly there. Sections 1-3 set the stage by surveying three other arguments for necessitism. I argue that none of them are persuasive, but I think it is illuminating to consider my argument in light of the others and vice versa. These interconnections should be of interest even to those who reject necessitism; of particular interest may be the new conception of validity proposed in section 5.

1 Combining logics

Following Williamson (2013), let *necessitism* be the claim expressed by the formula

NNE $\Box\forall x\Box\exists y(y = x)$

when $\Box$ is read as expressing the kind of 'metaphysical' necessity familiar from Kripke (1972) and the quantifiers are read as unrestricted. One sometimes hears that necessitism falls out of the combination of classical quantification theory and standard propositional modal logic. In this section I will explain one sense in which this is true and argue that it does not support a persuasive argument for necessitism.

By a *logic* I will mean a set of sentences of some formal language. Logics are usually specified in two steps. First, we write down some axiom schemata. Second, we define our logic to be the smallest set of sentences of our formal

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language that both contains every instance of any of the specified schemata and satisfies certain closure conditions. The closure conditions are sometimes called 'rules of inference', but we should not mistake them for prescriptions about how to reason: they are simply properties of sets. Call this way of specifying a logic *axiomatization*.

To take a familiar example, classical proposition logic can be axiomatized as the smallest set of sentences that contains all instances of any of the following three schemata

$$\varphi \to (\psi \to \varphi)$$

$$(\varphi \to (\psi \to \chi)) \to ((\varphi \to \psi) \to (\varphi \to \chi))$$

$$(\neg\varphi \to \neg\psi) \to (\psi \to \varphi)$$

and satisfies the closure condition that, for all sentences φ and ψ, if the set contains both φ and $\ulcorner \varphi \to \psi \urcorner$, then it also contains ψ. The usual notation for this sort of closure condition is:

MP If $\vdash \varphi$ and $\vdash \varphi \to \psi$, then $\vdash \psi$.

Both universal quantification over sentences of the language and corner quotes have been elided, and concatenation with the turnstile is used to abbreviate the claim that the sentence in question is a member the logic in question. We will adopt this notational convention in what follows.

The *logic of* some constants $\alpha_1, \ldots, \alpha_n$, which we shall denote $\vdash_{\alpha_1,\ldots,\alpha_n}$, is the set of sentences of the language under consideration that are *valid* when $\alpha_1, \ldots, \alpha_n$ are regarded as one's logical constants. For now we can remain neutral on the question of exactly how 'valid' is being used here; we will return to that question in section 5. However, I will assume that $\vdash_{\neg,\to}$ is classical propositional logic – i.e., that, relative the choice of $\neg$ and $\to$ as our logical constants, the logic just axiomatized is the set of all and only the valid sentences of our formal language. (I will use $\wedge, \vee, \exists, \neq$ and $\Diamond$ to abbreviate formulas containing $\neg, \to, \forall, =$, and $\Box$, in the familiar way.)

We now turn to logics of quantification and modality. It is widely held that the logic of boolean connectives, first-order quantification, and identity (i.e., $\vdash_{\neg,\to,\forall,=}$) includes all instances of the two schemata

=I $a = a$, where a is an individual constant

∃I $\varphi \to \exists x \varphi[x/a]$, where x is free for a in φ

and satisfies the closure condition

GEN If $\vdash \varphi$, then $\vdash \forall x\varphi[x/a]$, where x is free for a in φ.

It is also widely held that the logic of boolean connectives and a sentential operator $\Box$ expressing metaphysical necessity (i.e., $\vdash_{\neg,\to,\Box}$) satisfies the closure condition

RN If $\vdash \varphi$, then $\vdash \Box\varphi$.

These assumptions form the core of the following argument for the validity of NNE when boolean connectives, quantifiers, identity, and necessity are all taken as logical constants. The argument has six premises:

(i) $\vdash_{\neg,\to,\forall,=}$ includes all instances of =I and $\exists$I and is closed under MP.

(ii) $\vdash_{\neg,\to,\forall,=}$ is closed under GEN.

(iii) $\vdash_{\neg,\to,\Box}$ is closed under RN.

(iv) If $\vdash_{\neg,\to,\forall,=}$ is closed under GEN, then $\vdash_{\neg,\to,\forall,=,\Box}$ is closed under GEN.

(v) If $\vdash_{\neg,\to,\Box}$ is closed under RN, then $\vdash_{\neg,\to,\forall,=,\Box}$ is closed under RN.

(vi) For all φ, if $\vdash_{\neg,\to,\forall,=} \varphi$, then $\vdash_{\neg,\to,\forall,=,\Box} \varphi$.

The argument runs as follows, where a is an arbitrary individual constant of our language:

The Combination Argument

1. $\vdash_{\neg,\to,\forall,=} a = a$	from premise (i)
2. $\vdash_{\neg,\to,\forall,=} a = a \to \exists y(y = a)$	from premise (i)
3. $\vdash_{\neg,\to,\forall,=} \exists y(y = a)$	from 1, 2 and premise (i)
4. $\vdash_{\neg,\to,\forall,=,\Box} \exists y(y = a)$	from 3 and premise (vi)
5. $\vdash_{\neg,\to,\forall,=,\Box} \Box\exists y(y = a)$	from 4 and premises (iii) and (v)
6. $\vdash_{\neg,\to,\forall,=,\Box} \forall x\Box\exists y(y = x)$	from 5 and premises (ii) and (iv)
7. $\vdash_{\neg,\to,\forall,=,\Box} \Box\forall x\Box\exists y(y = x)$	from 6 and premises (iii) and (v)

Our conclusion 7 implies necessitism, since NNE cannot be valid in any sense without being true. This argument is the best way I know of arguing for necessitism on the grounds that it falls out of the natural way of combining classical first-order logic and standard propositional modal logic.[1]

The above argument is overkill because necessitism is overkill. Those who accept *contingentism* – the negation of necessitism – do so because they think that you and I are contingent beings: in particular, they think that '$\Box\exists y(y = \mathrm{JG})$' is false, and hence not valid in any sense. Since the argument for 5 above, if sound, immediately generalizes to establish the validity of this sentence, we may restrict our attention to that intermediate conclusion. Since premises (ii) and (iv) played no role in the argument for 5, we may ignore them in what follows.

Premise (vi) is secure: on any reasonable notion of validity a sentence cannot go from valid to invalid simply by considering more expressions as logical constants. Premise (iii) seems secure too. No counterexamples to it are suggested by the considerations that usually motivate philosophers to reject necessitism. Moreover, it is also widely held that $\vdash_{\neg,\rightarrow,\Box}$ is the logic S5, which satisfies RN. That leaves premises (i) and (v).

What reason is there to accept (v)? The mere fact that the logic of one set of expressions satisfies a certain closure condition doesn't imply that the logic of a superset of those expressions will also satisfy that condition. To take a trivial example, consider:

(*) If $\vdash \varphi$, then $\vdash \psi$.

$\vdash_{\neg}$ trivially satisfies (*), since it is the empty set, but $\vdash_{\neg,\rightarrow}$ clearly does not.[2] The proponent of (v) might appeal to the following disanalogy between (*)

[1]Most discussion of these issues, especially in the early literature on quantified modal logic, focusses not on NNE but on the schema CBF: $\exists x\Diamond\varphi \rightarrow \Diamond\exists x\varphi$. CBF was first derived by Barcan (1946) in a logic that is not closed under RN. Prior (1956) derives BF (the converse of CBF) assuming the validity of all instances $\exists$I and all theorems of the modal logic S5 and the closure of validities under RN and GEN. Kripke (1963) did the same for CBF with the weaker system K in place of S5. All of these derivations were in languages without identity. Once identity is added to the language, NNE and CBF are interderivable given very weak assumptions; see Williamson (2013, p. 44f).

[2]A slightly less trivial example: $\vdash_{\neg,\rightarrow}$ but not $\vdash_{\neg,\rightarrow,=}$ satisfies the condition that, if $\vdash Raa$, then $\vdash Raa \wedge \neg Raa$, since $\vdash_{\neg,\rightarrow}$, though not empty, contains no sentences of the form $\ulcorner Raa \urcorner$. (Note: this argument only works for languages without predicate abstraction, since otherwise $\vdash_{\neg,\rightarrow} (\lambda xy.\varphi \rightarrow \varphi)aa$.) Here is a less trivial example in a higher-order language with predicate abstraction. Define $\mathcal{T}$ to mean $(\lambda X.\forall p(Xp \rightarrow Xp))$, where p is a variable in sentence position. Then $\vdash_{\mathcal{T}}$ but not $\vdash_{\mathcal{T},\neg}$ satisfies the condition that, if $\vdash O\varphi$,

and RN. RN is central to our actual practice of axiomatizing modal theories. Most work in modal logic is concerned with logics that are *normal*, and closure under RN is part of the definition of normality. By contrast, (*) isn't useful for axiomatizing any non-trivial logic. Perhaps, then, the fact that RN is a theoretically fruitful closure condition on axioms is reason to think that $\vdash_{\neg,\rightarrow,\forall,=,\Box}$ satisfies it. In support of this claim, one might appeal to the fact that $\vdash_{\neg,\rightarrow,\forall,=,\Box}$ almost certainly satisfies MP and GEN, and not for some idiosyncratic reason but rather for the same reason that $\vdash_{\neg,\rightarrow,\forall,=}$ does.

My own preferred way of thinking about validity does indeed have the consequence that validities will continue to be closed under MP, RN, and something very close to GEN as more expressions are considered logical constants; see section 5. However, this conclusion depends on the details of an idiosyncratic way of thinking about validity, and so is of limited dialectical interest in the present context. For example, many deny that $\vdash_{\neg,\rightarrow,\Box,@}$ satisfies RN, where @ is a rigidifying 'actuality' operator: they claim that $\vdash_{\neg,\rightarrow,\Box,@} \varphi \leftrightarrow @\varphi$ even though $\nvdash_{\neg,\rightarrow,\Box,@} \Box(\varphi \leftrightarrow @\varphi)$. GEN also seems to fail for certain choices of logical constants. For example, one might argue that $\vdash_{\neg,\rightarrow,\forall,=,0,1} \neg(0 = 1)$, despite the fact that clearly $\nvdash_{\neg,\rightarrow,\forall,=,0,1} \forall x \forall y \neg(x = y)$.[3] The fact that a closure condition is commonly used to axiomatize the logic of some set of expressions is some evidence that it will hold of the logic of a given superset of those expressions, but the strength of such precedents is

then $\vdash OO\varphi$, where O is a schematic monadic sentential operator. And $\vdash_{\mathcal{T}}$ satisfies it non-vacuously, since it contains $\ulcorner(\lambda p.p)\mathcal{T}(\lambda p.p)\urcorner$ which is of the form $\ulcorner O\varphi \urcorner$.

[3]Here is a different argument for the same conclusion that does not rely on treating individual constants as logical constants. Let B be a sentential operator abbreviating 'S believes that ...'. Many philosophers think that $\vdash_{\neg,\rightarrow,B}$ contains all instances of the schema $B\varphi \rightarrow \neg B \neg \varphi$, in which case so must $\vdash_{\neg,\rightarrow,\forall,=,B}$, and hence $\vdash_{\neg,\rightarrow,\forall,=,B} BFa \rightarrow \neg B \neg Fa$. Yet many of these philosophers will deny that $\vdash_{\neg,\rightarrow,\forall,=,B} \forall x(BFx \rightarrow \neg B \neg Fx)$, and will therefore have to deny that $\vdash_{\neg,\rightarrow,\forall,=,B}$ is closed under GEN. The idea, following Kaplan (1968), would be that sentences involving quantification into 'believes' have truth conditions given by existential quantification over modes of presentation, in a way that allows $\ulcorner \forall x(x = a \rightarrow (BFx \rightarrow \neg B \neg Fx))\urcorner$ to be false while $\ulcorner BFa \rightarrow \neg B \neg Fa \urcorner$ is true. For example, they will think that 'If Lois believes that Superman flies, then Lois does not believe that Superman doesn't fly' is true, but 'For all x, if x is Superman, then if Lois believes that x flies, then Lois does not believe that x doesn't fly' is false, since there is someone (Superman) such that, under one mode of presentation (i.e., 'Superman'), Lois believes that he flies, yet under another mode of presentation (i.e., 'Clark'), Lois believes that he doesn't fly. Note that such views involve rejecting (i), since they entail the invalidity of $\ulcorner(BFa \wedge \neg B \neg Fa) \rightarrow \exists x(BFx \wedge \neg B \neg Fx)\urcorner$, which is an instance of ∃I.

unclear, and so is not a convincing way of motivating (v).[4]

Are there any other reasons to accept (v)? The only one that comes to mind is the view that sentences cannot be valid (relative to any choice of logical constants) unless they express necessary truths. We will revisit this claim in section 5. For now, it suffices to observe that, if one did accept this modal constraint on validity, then whatever considerations motivate contingentism immediately motivate denying the intermediate conclusion 3 above, and hence rejecting premise (i). So the Combination Argument does not constitute a dialectically powerful argument for necessitism. Contingentists will reject either premise (i) or premise (v), depending in part on what they think about relation between validity and necessity.

[4]One might object that the above argument proves too much, since it seems to lead to skepticism about whether even MP will continue to hold when we consider some new expressions as logical constants, yet surely we know that it will. I do not deny that we know this. But those who want to allow for failures of RN and GEN can allow that such knowledge is possible on the grounds that MP has a special status that both RN and GEN lack. Consider the following schematic injunction:

RMP From φ and $\ulcorner \varphi \rightarrow \psi \urcorner$, infer ψ.

On one natural interpretation of this injunction, it asserts that from the premises φ and $\ulcorner \varphi \rightarrow \psi \urcorner$ (which need not be logical truths in any sense), we may draw the conclusion ψ. It is a difficult question how exactly we should understand this normative claim, but however exactly it should be understood the following generalization suggests itself: if a schematic injunction $\mathcal{I}$ has the normative status typified by RMP, then the closure condition on sentences $\mathcal{C}$ that stands to $\mathcal{I}$ as MP stands to RMP will be satisfied by the logic of any set of expressions that includes all the expressions that are mentioned in $\mathcal{C}$ – i.e., in our notation, all those expressions that occur non-schematically after the turnstile. This generalization entails, as a special case, that the logic of any set of expressions that includes the material conditional will be closed under MP. Together with our knowledge of the normative status of RMP, it provides a principled basis for holding that MP has the robustness that we know it has. By contrast, no analogous argument can be used to establish that the logic of any set of expressions containing $\Box$ will be closed under RN, or that the logic of any set of expressions containing $\forall$ will be closed under GEN, because the injunctions that stand to RN and GEN as RMP stands to MP do not correspond to acceptable patterns of arguing from premises: from the non-logical premise ‘It is raining’, one clearly cannot conclude ‘Necessarily, it is raining’, and from the non-logical premise ‘Jones is happy’, one clearly cannot conclude ‘Everyone is happy’.

2 A direct argument

A distracting feature of the Combination Argument is that it was formulated entirely in terms of the validity of sentences. Not only is the notion of validity contested, but it is also besides the point: our question is not whether NNE is valid but simply whether necessitism is true.

This suggests a different strategy for necessitists: forget about the validity of sentences like $\ulcorner a = a \urcorner$ and $\ulcorner a = a \to \exists y(y = a) \urcorner$, and instead focus on the modal status of the propositions they express. This thought suggests the following direct argument for necessitism:

The Direct Argument

NI $\Box\forall x\Box(x = x)$

IE $\Box\forall x\Box(x = x \to \exists y(y = x))$

Therefore, $\Box\forall x\Box\exists y(y = x)$.

This argument has a number of virtues. It is uncontroversially valid. Its premises and conclusion are straightforward claims about modal reality, rather than claims about the validity of sentences of a formal language. And each of its premises enjoys intuitive support. How could anything possibly fail to be that very thing? And how could anything possibly be identical to itself without being identical to something – namely, itself?

Clearly the Direct Argument is not going to persuade any committed contingentists to reconsider their position. But the argument is diagnostically valuable nevertheless. For as we will see in what follows, the distinction between contingentists who reject NI and contingentists who reject IE is arguably the most important taxonomy of forms of contingentism. The crucial difference concerns the following schema, variously known as 'serious actualism' (Plantinga, 1983), 'the falsehood principle' (Fine, 1981) and the 'being constraint' (Williamson, 2013), according to which atomic predications necessitate there being the individuals of which they are predications:

BC $\Box\forall x_1 \ldots \Box\forall x_n\Box(Rx_1 \ldots x_n \to \exists y(y = x_i))$ for $1 \leq i \leq n$

Contingentists like Stalnaker (2012) who accept IE do so because they accept the more general BC; call them *negative contingentists*. Conversely, contingentists who reject BC will presumably accept NI too; call them *positive* *contingentists*. (The 'negative'/'positive' terminology is adopted from the literature on free logic.) That said, their motivation for regecting BC need not be a desire to reconcile contingentism with NI. For example, Bacon (2013) rejects BC for reasons having to do with the reference of empty names, and later I will argue that there are stronger reasons than NI for contingentists to reject BC.

3 Simplicity

A third argument for necessitism appeals to the simplicity of quantified modal logics containing NNE – in particular, those that include all instances of ∃I and are closed under RN. In comparison to contingentism-friendly logics that are closed under RN, and hence give up ∃I, Williamson writes:

> Without [∃I], the axiomatization of quantified modal logic becomes much harder; the required complications are greater than in the formal semantics, and completeness proofs are more convoluted. Such complications are a warning sign of philosophical error. (Williamson, 1998, p. 262)

Linsky and Zalta (1994) make a similar charge of comparative complexity. But it has little basis, as I will now explain.

Williamson's talk of 'quantified modal logic' in the above passage is best understood model theoretically, given his mention of completeness proofs. Let's restrict our attention to variable-domain Kripke models (with no accessibility relation) of a first-order modal language with identity. The logic determined by the class of such models in which NNE and all instances of ∃I are true – i.e., those in which every world has the same domain and every individual constant denotes an individual in that domain – is easily axiomatized by stitching together the standard axiomatizations of first-order logic with identity and propositional S5 in the way described in section 1. But the positive-contingentism-friendly logic determined by the class of variable domain Kripke models that validate NI (i.e., in which $\langle x, x\rangle$ is in the extension of the identity predicate at a world whenever x is in the domain of some world or is the denotation of some individual constant) can be axiomatized almost as easily: we simply replace of ∃I by the weaker

F∃I $\forall y(\varphi \rightarrow \exists x\varphi[x/y])$, where x is free for y in φ.

Things are only slightly more complicated in the case of the negative-contingentism-friendly logic determined by the class of variable domain Kripke models that validate IE (i.e., in which $\langle x, x \rangle$ is in the extension of the identity predicate at a world just in case x is in the domain of that world), whose axiomatization differs from the previous one by the addition of the schema

EE $Ra_1, \ldots, a_n \rightarrow \exists x(x = a_i)$ for $1 \leq i \leq n$,

and by the replacement the schema =I with the axiom

F=I $\forall x(x = x)$

(EE suffices for BC because in axiomatizing the logic we close it under GEN and RN.) These are hardly gross complications. The logic determined by the natural class of variable domain Kripke models (on either a positive or negative treatment of identity and other predicates) is not appreciably harder to axiomatize than the logic determined by the subclass of these models in which NNE and all instances of ∃I are true.[5]

Even if the aforementioned contingentism-friendly logics were significantly harder to axiomatize than the necessitism friendly-logic just mentioned, it is not clear why that should be considered evidence for necessitism. After all, in many cases (e.g. arithmetic) the logic determined by the natural class of models is incomplete, and so has no recursively specifiable axiomatization. If there is no harm in incompleteness in the case of arithmetic, why should the comparatively more tractable status of admitting only somewhat complicated axiomatizations be a problem in the case of first-order modal logic? Moreover, it is not clear that the logics we ought to be comparing in the first

[5]See Bacon (2013) for further discussion of these issues. It is true that in axiomatizing logics not containing all instances of ∃I we have to be sensitive to the fact that axiomatic strategies that are equivalent given ∃I may no longer be equivalent; see Fine (1983) and Williamson (2013, chapter 4). It is also true that, in modal logics that fail to include the B schema ($\varphi \rightarrow \Box\Diamond\varphi$), we may need to add a somewhat inelegant closure condition on axioms governing the interaction of quantifiers and modal operators (Hughes and Cresswell, 1996, p. 295). This point is relevant even if it does not arise in the case of metaphysical necessity (which is widely assumed to validate the B schema). For there may be yet stronger notions of necessity for which the B schema fails; see Bacon (unpublished) and Goodman (in preparation). Considerations of axiomatic complexity might then be used to argue that NNE was true when interpreted using this broader notion of necessity, in which case it would have to be true for metaphysical necessity too. I must leave further exploration of these considerations for another occasion.

place are the ones determined by the aforementioned classes of Kripke models. For suppose we adopt the Tarski-Williamson conception of validity mentioned in section 5. Then not only is the correct first-order modal logic (i.e., $\vdash_{\neg,\rightarrow,\forall,=,\Box}$) stronger than the logic determined by the class of variable-domain Kripke models of NNE and ∃I (since it counts $\exists x \exists y (x \neq y)$ as valid despite it not being true in all such models), but, as Fritz (2016) has explored in detail, its exact contours depend on the relative cardinality of possible worlds and individuals in surprisingly complicated ways. In this respect, then, it is not clear that things are so simple on the necessitist side of the ledger.

4 A new argument for necessitism

In this section I will give a new argument for necessitism. The argument deploys two unfamiliar connectives. The first is a dyadic sentential operator, which we shall write $\leq$. For formulas φ and ψ, we may read $\ulcorner \varphi \leq \psi \urcorner$ as $\ulcorner$that φ entails that $\psi\urcorner$. (This is merely a pronunciation recipe, not an attempt to define $\leq$ in terms of the vernacular meaning of 'entails'.) The second connective takes monadic predicates rather than sentences as arguments. We will write it $\leq$ also, letting the syntactic category of its flanking expression indicate which connective we are using on a given occasion. For predicates F and G, we may read $\ulcorner F \leq G \urcorner$ as $\ulcorner$being F entails being $G\urcorner$. Shortly we will consider ways in which these connectives might be understood so as to make true the seven principles below. But let's get the principles on the table first:

REFLEXIVITY
$\varphi \leq \varphi$

TRANSITIVITY
$(\varphi \leq \psi) \wedge (\psi \leq \chi) \rightarrow (\varphi \leq \chi)$

APPLICATION
$F \leq G \rightarrow Fa \leq Ga$

REDUCTION
$(\lambda x.\varphi)a \leq \varphi[a/x]$, where a is free for x in φ

ABSTRACTION
$\varphi[a/x] \leq (\lambda x.\varphi)a$, where a is free for x in φ

ADJUNCTION[6]
$\varphi \leq \forall x\psi \leftrightarrow (\lambda x.\varphi) \leq (\lambda x.\psi)$, where x is not free in φ

NECESSITATION
$\varphi \leq \psi \rightarrow \Box(\varphi \rightarrow \psi)$

As usual, for any formula φ, $\ulcorner(\lambda x.\varphi)\urcorner$ is a monadic predicate that takes expressions of the same syntactic type as x as arguments and in which all occurrences of x are bound. As it occurs in predicate position, one might read $\ulcorner(\lambda x.\varphi)\urcorner$ as $\ulcorner$is such that $\varphi[\text{it}/x]\urcorner$, where all relevant occurrences of 'it' are anaphoric on the predicate's argument. For example, we might read '(λx. Napoleon was from x)France' as 'France is such that Napoleon was from it'. When such predicates occur as arguments to $\leq$, 'you' sounds better than 'it' for some reason, as in 'being such that you are a square entails being rectangular'. However, as with $\leq$, the interpretation of the formalism is not hostage to the meaning of such quasi-English locutions.

The seven principles above imply that any arbitrary individual a is a necessary being. Here is why:

The Adjunction Argument

1. $\forall x\exists y(y = x) \leq \forall x\exists y(y = x) \rightarrow (\lambda x.\forall x\exists y(y = x)) \leq (\lambda x.\exists y(y = x))$ (ADJUNCTION, left-to-right)
2. $(\lambda x.\forall x\exists y(y = x)) \leq (\lambda x.\exists y(y = x))$ (1, REFLEXIVITY)
3. $(\lambda x.\forall x\exists y(y = x))a \leq (\lambda x.\exists y(y = x))a$ (2, APPLICATION)
4. $\forall x\exists y(y = x) \leq (\lambda x.\forall x\exists y(y = x))a$ (ABSTRACTION)
5. $(\lambda x.\exists y(y = x))a \leq \exists y(y = a)$ (REDUCTION)
6. $\forall x\exists y(y = x) \leq \exists y(y = a)$ (3,4,5, TRANSITIVITY)
7. $\Box(\forall x\exists y(y = x) \rightarrow \exists y(y = a))$ (6, NECESSITATION)
8. $\Box\exists y(y = a)$ (7, uncontroversial modal principles)

[6] The principle is so-named because, in the terminology of category theory, it says that universal generalization, thought of as an operation taking properties to propositions, is the 'right adjoint' of vacuous predicate-abstraction, thought of as an operation taking propositions to properties; existential generalization is then the 'left adjoint' of vacuous predicate-abstraction. See Dorr (2014).

The argument is clearly valid, and its conclusion is tantamount to necessitism for reasons mentioned in section 1. The question is whether there is a way of interpreting $\leq$ so as to make the argument interesting. For example, suppose we interpret $\varphi \leq \psi$ as $\Box(\varphi \to \psi)$. NECESSITATION is then true by definition. The problem is that the needed instance of ABSTRACTION becomes definitionally equivalent to a first-order modal formula that negative contingentists are already happy to reject. As for positive contingentists, defining $F \leq G$ as $\Box\forall x\Box(Fx \to Gx)$ makes the needed instance of ADJUNCTION definitionally equivalent to a first-order modal formula they are already happy to reject; defining it instead as $\Box\forall x(Fx \to Gx)$ does the same for the needed instance of APPLICATION. The Adjunction Argument is therefore not dialectically effective if we stipulate that entailment be understood in modal terms.

A different approach would be to invoke a theory on which propositions are structurally isomorphic to sentences, and to interpret $\varphi \leq \psi$ as the claim that the proposition that $\varphi \to \psi$ is isomorphic to a valid sentence. (For some work in this direction, see Fine (1990).) The problem (setting aside general worries about structured propositions) is that, by making entailments parasitic on the validity of sentences, it renders the Adjunction Argument dialectically impotent for the same reason that the Combination Argument was. Those contingentists who want to say that '$\exists x(x = a)$' is valid despite expressing a contingent truth will happily reject NECESSITATION on the proposed interpretation of entailment. The remaining contingentists will fall into the same taxonomy as before; the negative contingentists will happily reject ABSTRACTION and the positive contingentists will happily reject either ADJUNCTION or APPLICATION depending on how we choose to define the entailment connective that takes predicates as arguments.

A third approach would be to take the relevant notion of entailment as given by examples. In addition to easy cases, such as that conjunctions should entail their conjuncts, one might say that being red entails being colored, or that that I know that it is raining entails that I believe that it is raining. One might also appeal to the 'part of what it is' idiom, as in 'part of what it is to be red is to be colored', so as to screen off various epistemic readings of 'entails'. It is unclear whether such examples do enough to isolate a serviceable notion of entailment. The problem concerns ADJUNCTION. As will emerge, there is no simple argument for that principle – rather, it must be motivated abductively, in terms of the strength it lends to theories in which it figures. So even assuming that there is a notion of entailment in the vicinity of the above examples, in the absence of a theoretical role for the notion it is not obvious why ADJUNCTION formulated in terms of it should be an attractive principle, especially if the choice is between it and contingentism.

My own approach will be to define the two notions of entailment in terms of disjunction and a corresponding pair of notions of 'identification', which we will write $\equiv$. That is, we define $\varphi \leq \psi$ as $(\varphi \vee \psi) \equiv \psi$ and $F \leq G$ as $(\lambda x.Fx \vee Gx) \equiv G$. In contrast to the first two proposals, our grip on identifications is sufficiently independent of our judgments about modal matters that the Adjunction Argument, so formulated, can be dialectically effective. And in comparison to the third proposal, the appeal of ADJUNCTION becomes much clearer when entailment is understood in terms of identifications.

A number of authors have recently argued that connectives analogous to the identity predicate but that take formulas or predicates as arguments are both intelligible and theoretically fruitful; see Rayo (2013), Goodman (forthcoming), and especially Dorr (this volume). We can get a grip on such notions of identification in a number of ways. One is that the notions should obey principles analogous to those governing identity: that is, they should be reflexive and support some version of Leibniz's Law, so that true identifications license the intersubstitution of the flanking sentences/predicates in non-opaque contexts (where opaque contexts are typified by the failure of true identities involving proper names to license the intersubstitution of those names in those contexts). Another strategy for isolating the intended notions is to focus on certain uses of the constructions 'to be F is to be G' and 'for it to be the case that φ just is for it to be the case that ψ'. A third strategy is to explicitly define notions of identification using higher-order quantification: just as the first-order identity predicate can be defined in terms of second-order indiscernibility ($x = y$ is defined as $\forall F(Fx \leftrightarrow Fy)$), we can use third-order quantification to define notions of identification whose arguments are sentences or predicates (i.e., $\varphi \equiv \psi$ is defined as $\forall O(O\varphi \leftrightarrow O\psi)$, where O is a variable of the same syntactic category as a monadic sentential operator like negation, and $F \equiv G$ is defined as $\forall X(XF \leftrightarrow XG)$, where X is a variable that takes a monadic predicate of individuals as an argument, in the way that first-order quantifiers do when they are treated as 'second-level predicates' following Frege.) Rather than defend this notion of identification further here, I will simply work with it and consider how the Adjunction Argument looks when entailment is understood in terms of identification and disjunction.

One advantage of this interpretation of the Adjunction Argument is that NECESSITATION becomes uncontroversial, though not true immediately by definition: $\varphi \leq \psi$ is definitionally equivalent to $(\varphi \vee \psi) \equiv \psi$, from which $\Box(\psi \to \psi) \leftrightarrow \Box((\varphi \vee \psi) \to \psi)$ follows by the analogue of Leibniz's Law for $\equiv$; $\Box(\varphi \to \psi)$ then follows by uncontroversial propositional modal reasoning.

A more controversial principle given the present interpretation of entailment is REFLEXIVITY, which becomes the claim that disjunction is idempotent: for it to be the case that φ just is for it be the case that φ or φ. Since sentences are not disjuncts of themselves, this principle will be denied by those who think that reality is structured in the manner of the sentences we use to talk about it. I will not consider such structured views here; in Goodman (forthcoming) I argue that the natural ways of developing them lead to contradiction. Still, I do not think that the idempotence of disjunction is sacrosanct. For example, Dorr (this volume) develops an attractive and powerful 'no circularity'-theory of reality's fineness of grain that has a wide range of non-trivial models and is inconsistent with the idempotence of disjunction. So he would have to reject the Adjunction Argument on the interpretation we are now considering. However, his framework supports a close variant of the argument, which for reasons of space I will confine to a footnote.[7]

[7] Dorr's theory is formulated in the λI-calculus, in which $\ulcorner(\lambda x.\varphi)\urcorner$ is well-formed only when x occurs free in φ. So the first thing we need to do is reformulate ADJUNCTION so that it doesn't require vacuous predicate abstraction, which can be done as follows:

> ADJUNCTION*
> $\varphi \leq \forall x\psi \leftrightarrow (\lambda x.\varphi \vee \psi) \leq (\lambda x.\psi)$, where x occurs free in ψ but not in φ.

We can then run the following variant of the Adjunction Argument:

1* $\forall x\exists y(y = x) \leq \forall x\exists y(y = x) \leftrightarrow (\lambda x.\forall x\exists y(y = x) \vee \exists y(y = x)) \leq (\lambda x.\exists y(y = x))$ (ADJUNCTION*, left-to-right)

2* $(\lambda x.\forall x\exists y(y = x) \vee \exists y(y = x)) \leq (\lambda x.\exists y(y = x))$ (1*, REFLEXIVITY)

3* $(\lambda x.\forall x\exists y(y = x) \vee \exists y(y = x))a \leq (\lambda x.\exists y(y = x))a$ (2*, APPLICATION)

4* $(\forall x\exists y(y = x) \vee \exists y(y = a)) \leq (\lambda x.\forall x\exists y(y = x) \vee \exists y(y = x))a$ (ABSTRACTION)

5* $(\lambda x.\exists y(y = x))a \leq \exists y(y = a)$ (REDUCTION)

6* $(\forall x\exists y(y = x) \vee \exists y(y = a)) \leq \exists y(y = a)$ (3*,4*,5*, TRANSITIVITY)

7* $\Box((\forall x\exists y(y = x) \vee \exists y(y = a)) \to \exists y(y = a))$ (6*, NECESSITATION)

8* $\Box\exists y(y = a)$ (7*, uncontroversial modal principles)

Note that 4*, unlike 4, does not involve vacuous predicate abstraction.

(*footnote continued on next page*)

By contrast, proponents of coarse grained conceptions of reality usually will accept the idempotence of disjunction. In particular, it is a commitment of *Booleanism* – the view, roughly, that φ can be identified with ψ whenever "$\varphi \leftrightarrow \psi$" is a theorem of classical propositional logic, with analogous identifications involving predicates too; see Dorr (this volume, section 7) for a precise statement of the view. Booleanism is an extremely strong and simple theory, and as such it ought to be taken very seriously. It also makes ADJUNCTION into a very powerful principle. This is because (reifying freely) ADJUNCTION pins down which propositions entail which generalizations, by providing necessary and sufficient conditions for such entailments in terms of relations of disjunction and identification among properties. Since Booleanism implies that entailment (i.e., disjunctive containment) is not only reflexive but also anti-symmetric (i.e., mutual entailment implies identity), propositions entailed by the same propositions must be identical. In this way ADJUNCTION *uniquely characterizes the semantic contribution of universal*

The next step is to find an interpretation of $\leq$ that makes the above argument compelling in Dorr's setting. First, we define a pair of notions of 'logical equivalence' $\approx$ in terms of conjunction, disjunction and identification: $\varphi \approx \psi$ is defined as $(\varphi \wedge \psi) \equiv (\varphi \vee \psi)$ and $F \approx G$ is defined as $(\lambda x.Fx \wedge Gx) \equiv (\lambda x.Fx \vee Gx)$. Next, we interpret $\leq$ not in terms of $\equiv$ and disjunction, but in terms of $\approx$ and disjunction. So interpreted, Dorr's models validate REFLEXIVITY – although we cannot identify φ and $\varphi \vee \varphi$, we can identify their conjunction with their disjunction. Furthermore, his models validate TRANSITIVITY, APPLICATION, REDUCTION, ABSTRACTION, and ADJUNCTION* on this interpretation of entailment. The interpretation also vindicates NECESSITATION: from $\varphi \leq \psi$, we get $(\varphi \vee \psi) \approx \psi$ by the definition of $\leq$, then $((\varphi \vee \psi) \wedge \psi) \equiv ((\varphi \vee \psi) \vee \psi)$ by the definition of $\approx$, then $\Box(((\varphi \vee \psi) \wedge \psi) \to ((\varphi \vee \psi) \wedge \psi)) \leftrightarrow \Box(((\varphi \vee \psi) \vee \psi) \to ((\varphi \vee \psi) \wedge \psi)))$ by Leibniz's Law for $\equiv$; and finally $\Box(\varphi \to \psi)$ by propositional modal reasoning.

The fact that Dorr's models validate ADJUNCTION* on the present interpretation of entailment shows that the principle is tenable in his setting. But it is not much of an argument for that principle; one could easily modify his model construction so as to make it friendly to contingentism. However, just as Booleanism implies that propositions entailed by the same propositions can be identified (see the next paragraph in the main text), provided 'entailment' is understood in terms of disjunction and identification, Dorr's theory likewise implies that propositions entailed by the same propositions are logically equivalent, provided 'entailment' is understood in terms of logical equivalence and disjunction. In this way, just as ADJUNCTION uniquely pins down the semantic contribution of quantification given Booleanism, ADJUNCTION* likewise uniquely pins down the logical contribution of quantification given Dorr's theory, and is thereby recommended by its strength in the context of that theory.

quantification.[8] This is the sense in which, given Booleanism, ADJUNCTION is a strong and thereby attractive principle.

(This is not to say that the appeal of ADJUNCTION is exhausted by its strength. It is also recommended by its elegance and perhaps enjoys a degree of pretheoretical plausibility too. Its naturalness is further revealed by its connections to ∃I and GEN to be discussed in section 5.)

It is worth noting that the most systematic developments of contingentism have been in a Booleanist setting (e.g., Fine (1977) and Williamson (2013), although see Fritz and Goodman (2016, section 3.4) for a way in which non-Booleanists might reinterpret such theories) and that many contingentists are in fact Booleanists (e.g., Stalnaker (2012), Rayo (2013), and Bacon (forthcoming)). So the appeal to Booleanism in the above argument for ADJUNCTION does not beg the question against contingentism.

Moreover, that argument does not require the full strength of Booleanism. It only relied on the assumption that entailment is reflexive and anti-symmetric. One can accept these principles of granularity while rejecting other features of Booleanism. To give a concrete example of how this might go, let me briefly sketch my own preferred view of reality's fineness of grain. According to my view, claims about reality vary along two dimensions: their 'logical contents', which form a Boolean algebra with respect to negation, conjunction, and disjunction, and their 'objectual contents' – i.e., the pluralities individuals they are about. Booleanism is false because claims with the same logical content can differ in which individuals they are about. For example, Fa and $Fa \wedge (Gb \rightarrow Gb)$ have the same logical content, but only the latter will be about b, assuming $a \neq b$ and F is not about b. But like Booleanism, my view implies that entailment is reflexive (since φ and $\varphi \vee \varphi$ have the same logical content and are about the same individuals) and anti-symmetric (since $\varphi \vee \psi$ and $\psi \vee \varphi$ have the same logical content and are about the same individuals).[9]

[8]This fact uniquely pins down the meaning of the universal first-order quantifier if we assume the principle of *functionality* according to which conditions that yield the same proposition for all arguments must be the same condition (where we think of quantifiers as conditions on properties of individuals). Such a uniqueness result would allow us to eliminate first-order quantification in favor of third-order quantification – and, more generally, nth order quantification in terms of n+2th order quantification – by treating first-order quantifiers as indefinite descriptions of conditions satisfying the universal closure of ADJUNCTION, and then contextually eliminating these descriptions using existential quantification in the manner of Russell (1905).

[9]Goodman (unpublished) shows how the reflexivity and anti-symmetry of disjunctive containment is also a consequence of a natural way of developing the view that claims

So the view still supports the above abductive argument for ADJUNCTION. Since the most natural way of developing the view also vindicates TRANSITIVITY, APPLICATION, ABSTRACTION, and instances of REDUCTION such as 5 involving non-vacuous predicate abstraction (i.e., in which the variable bound by λ occurs free in the embedded formula; see footnote 7), that abductive argument, together with an aboutness-theoretic account of reality's granularity, jointly support the Adjunction Argument for necessitism.[10]

In sum: given independently attractive theories of reality's fineness of grain, we can define notions of entailment in terms of identification and boolean operations that make the needed instances of REFLEXIVITY, TRANSITIVITY, APPLICATION, REDUCTION, ABSTRACTION, and NECESSITATION true and make ADJUNCTION attractively strong. In this way, these theories of granularity support an abductive argument for necessitism. I should emphasize that, although ADJUNCTION is attractively simple, the primary motivation for it is not its simplicity but its strength. Just as when axiomatics is at issue contingentists can replace ∃I with the not much more complicated F∃I, as discussed section 3, so too in the case of the metaphysics of quanification it is natural for contingentists who accept theories of granularity friendly to ADJUNCTION to instead accept the not much more complicated principle:

FREE ADJUNCTION
$\varphi \leq \forall x\psi \leftrightarrow (\lambda x.\varphi) \leq (\lambda y.\forall x(y = x \rightarrow \psi))$, where x is not free in φ.

FREE ADJUNCTION is not much more *complicated* than ADJUNCTION. But it is significantly *weaker*. To see why, let us assume that $(\lambda y.\forall x(y = x \rightarrow (Fx \rightarrow$

about reality differ along a dimension of 'subject matter' in addition to a dimension of logical content.

[10]In unpublished work I develop this theory of aboutness in much greater detail. Note that, in this setting, the above argument would not go through on a conjunctive definition of $\varphi \leq \psi$ as $\varphi \equiv (\varphi \wedge \psi)$, since 4 is false with $\leq$ so interpreted. That said, the Adjunction Argument can be reformulated using the conjunctive notion of entailment. We need a principle like ADJUNCTION* from footnote 7 so we can avoid using vacuous predicate abstraction. Unfortunately, ADJUNCTION* is not tenable, since the right-to-left direction can fail in the aboutness setting on a conjunctive interpretation of $\leq$; let φ be a contradiction about strictly fewer individuals than $\forall x\psi$ is about. Luckily, the dual principle

ADJUNCTION**
$\exists x\psi \leq \varphi \leftrightarrow (\lambda x.\psi) \leq (\lambda x.\varphi \wedge \psi)$, where x occurs free in ψ but not in φ

is tenable in that setting on the conjunctive interpretation. Reformulating the argument from footnote 7 in terms of ADJUNCTION** is left as an exercise for the reader.

ψ))) and $(\lambda y.\forall x(Fx \to (y = x \to \psi)))$ are mutually entailing (which is a consequence of Booleanism, my aboutness theory, and most other theories that are coarse-grained enough to be relevant for present purposes). The transitivity of entailment in predicate position then allows us to substitute the latter for the former in the relevant instances of FREE ADJUNCTION, yielding

> RESTRICTED FREE ADJUNCTION
> $\varphi \leq \forall x(Fx \to \psi) \leftrightarrow (\lambda x.\varphi) \leq (\lambda y.\forall x(Fx \to (y = x \to \psi)))$, where x is not free in φ.

That is, if FREE ADJUNCTION holds for the unrestricted quantifier, then given very weak assumptions it holds for all restricted quantifiers too, and thereby grossly fails to pin down the behavior of the unrestricted quantifier in anything like the way that ADJUNCTION does. (Dorr (2014) explains why we should like to be able to pin down the behavior of our quantifiers in this way.) By contrast, ADJUNCTION clearly fails if our quantifiers are read as restricted by any non-universal condition F, assuming entailment is interpreted in a way that validates REFLEXIVITY, since line 2 of the Adjunction Argument then becomes $(\lambda x.\forall y(Fx \to \exists y(Fy \wedge y = x))) \leq (\lambda x.\exists y(Fy \wedge y = x))$, which is false, since absolutely everything satisfies the first condition but every non-F fails to satisfy the latter condition.

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I have argued that, conditional on some albeit controversial principles about reality's fineness of grain, we ought to accept REFLEXIVITY, TRANSITIVITY and ADJUNCTION on an interpretation of entailment as disjunctive containment. As we have seen, NECESSITATION is uncontroversial on this interpretation. And APPLICATION is equally secure given ABSTRACTION and the assumption that disjunctions are entailed by their disjuncts.[11] So it remains only to consider REDUCTION and ABSTRACTION.

There are three main reasons why one might reject REDUCTION. One is the idea that distinctions in reality are structured in a way that reflects the syntax of the sentences we use to express them; we have already set such

[11] Suppose $F \leq G$; i.e., that $G \equiv (\lambda x.Fx \vee Gx)$. By REFLEXIVITY we have $Ga \leq Ga$. By Leibniz's law for $\equiv$ we get $(\lambda x.Fx \vee Gx)a \leq Ga$. ABSTRACTION gives us $(Fa \vee Ga) \leq (\lambda x.Fx \vee Gx)a$, so by TRANSITIVITY we have $(Fa \vee Ga) \leq Ga$. Assuming disjunctions are entailed by their disjuncts, we have $Fa \leq (Fa \vee Ga)$. A final application of TRANSITIVITY yields the desired conclusion that $Fa \leq Ga$.

views aside.[12] The second concerns the vacuous case in which x does not occur free in φ; one might then worry that $(\lambda x.\varphi)a$ and $\varphi[a/x]$ could differ in that only the former would be about a, precluding it from being a disjunct of the latter. I am sympathetic to this objection, but it does not threaten the restriction of REDUCTION to non-vacuous cases, which is all we need for the Adjunction Argument. A third reservation concerns propositional attitude verbs. Following Quine (1956) and Kaplan (1968), one might think that, in the Superman story, (λx. Lois knows that x flies)Clark even though Lois does not know that Clark flies, since she knows that Clark flies under some relevant mode of presentation, but not the one associated with 'Clark'; see Yalcin (2015) for a recent discussion. It is important to distinguish this view from the weaker one mentioned in footnote 3, according to which in order to give the right truth conditions for sentences involving quantification into the scope of propositional attitude verbs we need to existentially generalize over modes of presentation. Even if the latter view were correct, it is not obvious that variable binding as opposed to quantification is what induces such existential generalization. From a technical perspective, it is more natural to blame quantifiers than to blame mere variable binders, and doing so allows us to reconcile Kaplan-style truth conditions for quantified attitude ascriptions with REDUCTION; see Bacon and Russell (unpublished). At any rate, the Superman example suggests at most that REDUCTION may need to be restricted to cases in which x occurs free in φ only in non-opaque contexts. That restriction would not threaten the Adjunction Argument, which uses an instance of REDUCTION in which x occurs free only in extensional contexts.

Of these three concerns about REDUCTION, the first applies in equal measure to ABSTRACTION, the third applies in at most equal measure to ABSTRACTION (depending on whether or not we require the mode of presentation associated with an individual constant to always be in the domain of modes of presentation relevant to giving truth conditions for *de re* atti-

[12] A different objection to REDUCTION associated with the aforementioned structured picture is that we should distinguish John loves John from ($\lambda x.x$ loves x)John on the grounds that someone could believe the former but not the latter by thinking of John under different modes of presentation *qua* lover and *qua* beloved. In reply, no such distinction is required by the idea that some claim in the vicinity of John loves John is less easily believed on account of John occurring in it only once, since we can identify that claim with $\exists x(x =$ John and x loves $x)$. See Dorr (this volume) for a framework in which one can draw such distinctions without rejecting REDUCTION.

tude ascriptions), and the second does not apply (since both $(\lambda x.\varphi)a$ and $(\lambda x.\varphi)a \vee \varphi[a/x]$ are about a if either is). However, while negative contingentists – i.e., contingentists who accept the principle BC from section 2 – may be happy to accept REDUCTION, they cannot accept ABSTRACTION on any interpretation of entailment that supports NECESSITATION, since they reject $\Box(\forall x \exists y(y = x) \to (\lambda x.\forall x \exists y(y = x))a)$, and hence must reject step 4 of the Adjunction Argument. (This rejection cannot be confined to cases of vacuous predicate abstraction, since for any purported contingent being a they must likewise reject $\neg\exists y(y = a) \leq (\lambda x.\neg\exists y(y = x))a$ on the present interpretation, since they will reject the corresponding strict conditional $\Box(\neg\exists y(y = a) \to (\lambda x.\neg\exists y(y = x))a)$.)

In the remainder of this section I will argue that we should reject negative contingentism. For one thing, it appears to be inconsistent with the truth of the English sentences contingentists usually use to articulate and motivate their view. This is because in discussing modal matters we usually use adverbs and adverbial phrases like 'not' and 'could have' that by modifying the copula allow us to form complex predicates from simpler ones, rather than using sentential operators like 'it is not the case that' and 'it could have been the case that'. Even Williamson, who is the most fastidious writer on these matters, writes that 'we can capture contingentism in the negation of (NNE): possibly something is possibly nothing' (Williamson, 2010, p. 666) and that, for the reasons mentioned in section 1, contingentists will think that 'there is actually something that could have been nothing' (Williamson, 2013, p. 1). But 'is nothing' is a predicate of English. Suppose we instantiate BC with it, yielding a sentence we might naturally pronounce '(Necessarily) for all x, necessarily, if x is nothing, then there is something identical to x'. This claim is false provided 'is nothing' is interpreted so as to make true both 'For all x, necessarily, if x is nothing, then there is nothing identical to x' and 'For some x, possibly, x is nothing', which is how it must be interpreted in order for judgments expressed using it to motivate contingentism in the first place. It seems, then, that ordinary expressions of contingentist ideas directly contradict negative contingentism.

There are two ways that negative contingentists might respond to this charge. One is to bite the bullet and say that such ordinary judgments are incompatible with BC and are hence mistaken. The challenge for this view is to explain what then motivates contingentism, if these ordinary judgments do not, and, more challengingly, why contingentists should accept BC given that it convicts ordinary speakers of error. Notice that this view not only convicts contingentists of error when they offer metaphysical sounding claims to support their contingentism, but also convicts everyone of error in accepting such apparent banalities as ‘If my parents had never met, then I would not have been born’, since ‘would not have been born’ is a predicate of English! (Dorr (this volume, section 5) gives a more careful version of this argument.)

Alternatively, negative contingentists might claim that they are understanding ‘predicates’, in the sense of those expressions that can be used to instantiate BC, in a special way that excludes expressions like ‘is nothing’. For example, in the paper that has become the *locus classicus* of negative contingentism, Stalnaker (1977, p. 339) writes:

> It is also not implausible, I think, to say that *nonexistent* is not a predicate. Existence *is* a predicate – one that applies to everything there is – but statements that seem to ascribe the predicate of nonexistence to something are really denials of statements ascribing the predicate of existence.

It is hard to tell from this passage exactly how Stalnaker is thinking about the distinction between real predicates like ‘exists’ and merely apparent predicates like ‘is nonexistent’. It recalls the view of Fine (1985), according to which BC holds of all predicates that express metaphysically unanalyzable properties or relations, although such a notion of metaphysical analyzability is rather un-Stalnakerian. At any rate, we can set aside this demarcation question in favor of the following more relevant one: if not all predicates of English yield true substitution instances of BC, why think that all predicates formed by predicate abstraction yield true substitution instances of BC?

I suspect one reason why many negative contingentists find instances of BC like $\Box((\lambda x.\neg\exists y(y = x))a \rightarrow \exists y(y = a))$ plausible is that they tend to subvocalize (and often speak and write) expressions of the form $\ulcorner(\lambda x.\varphi)\urcorner$ as $\ulcorner$is an x such that $\varphi\urcorner$. This pronunciation scheme has two advantages over the one I suggested at the beginning of this section: (i) it yields somewhat less stilted English glosses, and (ii) it allows for a uniform pronunciation of λ-terms whether they occur in predicate position or as arguments to $\leq$ (compare $\ulcorner a$ is an x such that $\varphi\urcorner$ and $\ulcorner$being an x such that φ entails being $G\urcorner$), whereas the scheme I suggested required using a third-person pronoun to pronounce λ-terms in predicate position and a second-person pronoun to pronounce λ-terms as arguments to $\leq$. But while these conveniences make it understandable that we would pronounce λ-terms using the indefinite article, and indeed the good-standing of this practice is vindicated by necessitism, on the assumption of contingentism this pronunciation scheme contaminates our judgments, since it is uncontroversial that nothing could not have been *an* x such that $\neg\exists y(y = x)$ since being *an* anything requires being something. For this reason it should be uncontroversial and unsurprising that there is *one* way of reading the λ-notation on which the BC is valid for the predicates so-formed; primitive notation can always be interpreted in a range of ways. This observation does not answer the question of whether contingentists should think that λ-terms validate BC on their *intended* interpretation.

The answer to this question depends on what pretheoretical practice anchors our use of λ-terms. I claim it is our practice of using open formulas to introduce new expressions by explicit definition. Both in ordinary technical writing and when teaching beginning logic students to formalize natural language arguments in a first-order language by using dictionaries, we introduce new meaningful predicates into our language using open formulas as definitions. We do so with the understanding that when we use such a predicate (in predicate position) the resulting atomic formula will be intersubstitutable with the formula that result from replacing the variables in the predicate's definition with the predicate's arguments on that occasion.[13] The use of at least non-vacuous predicate abstraction systematizes this pre-existing practice; predicate abstraction is in effect a convenient notation for *ad hoc* definition. As such, we should be able to infer all non-vacuous instances of ABSTRACTION and REDUCTION from the corresponding instances of REFLEXIVITY, since that is how definitions work. It is this point – not judgments about my necessary self-identity or about my standing in psychological or referential relations to non-beings – that refutes negative contingentism.[14]

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My discussion has focused on a particular understanding of entailment as disjunctive containment, but let us now take a step back and consider the Adjunction Argument more schematically. We have good reason to think that

[13] To be clear, that this is how definitions work is not something we can make true by stipulation, but it is a truth to which we are sensitive and that is thereby reflected in our linguistic practice. As usual, we may have to restrict such intersubstitution to non-opaque contexts.

[14] Versions of this argument for ABSTRACTION are given by Fine (1985), Dorr (this volume, section 6) and Kripke (2005, p. 1025), who attributes this understanding of predicate abstraction to Russell, as does Hodes (2015). For some more technical reasons to be dissatisfied with negative contingentism, see Fritz and Goodman (2016).

reality supports some notion of entailment that makes all (or at least the relevant) instances of REFLEXIVITY, TRANSITIVITY, APPLICATION, REDUCTION and NECESSITATION true and is such that mutual entailment is an important kind of metaphysical equivalence. Many contingentists agree, if for no other reason than that they are Booleanists, so this starting point does not beg the question against them. The negative contingentists among them respond by rejecting ABSTRACTION; that is their mistake. The positive contingentists among them forsake ADJUCTION; their theory of the metaphysics of quantification is highly unconstrained. The resulting position, though coherent, is comparatively unattractive. Necessitism is a better way.

5 Entailment and validity

I want to close by drawing out some connections between the Combination Argument and the Adjunction Argument. First, I will present a new way of thinking about the validity of sentences in terms of entailments in reality, drawing heavily on Dorr (2014). I will then show how, given this conception of validity, (i) the left-to-right direction of ADJUNCTION guarantees that the logic of any set of expressions containing boolean connectives and the universal quantifier will include all instances of ∃I, and (ii) the right-to-left direction of ADJUNCTION guarantees that the logic of any set of expressions containing the universal quantifier will be closed under

GEN* If $\vdash \varphi$, then $\vdash \forall x\varphi[x/a]$, where a is a non-logical constant and x is free for a in φ.

(The fact that the right-to-left direction of ADJUNCTION was not needed to establish the conclusion of the Adjunction Argument corresponds to the fact that the closure of $\vdash_{\neg,\rightarrow,\Box,\forall,=}$ under GEN* was not required to establish the validity of that conclusion – i.e., it was not invoked in the argument for step 5 of the Combination Argument.) For ease of exposition, I will freely talk of propositions and properties as a way of conveying in English claims which are officially formulated using quantification into sentence and predicate positions.

So far we have only used the apparatus of predicate abstraction to form monadic predicates that take individual constants and variables as arguments. But the apparatus is much more flexible. In general, for any pairwise-distinct variables $v_1, \ldots, v_n$ of whatever syntactic types, $\ulcorner(\lambda v_1 \ldots v_n.\varphi)\urcorner$ will be a predicate in which all of those variables are bound and that takes n arguments of the respective syntactic types of those variables. I will take for granted that identifications make sense not only in the case of pairs of formulas or pairs of monadic predicates of individuals, but also in the case of any pair of predicates of the same syntactic type; see Dorr (this volume). This allows us to define a corresponding family of notions of being 'tautologous', which we will notate $\top$, in terms of being identifiable with one's own self-implication: $\top\varphi$ is defined as $(\varphi \to \varphi) \equiv \varphi$, $\top F$ is defined as $(\lambda x.Fx \to Fx) \equiv F$, etc.

For any sentence φ of our language, let f_φ be some arbitrary one-to-one function that maps every constant α that occurs in φ to a variable of the same syntactic type that is free for α in φ. For any set of constants C and sentence φ, let $\Sigma(\varphi, C)$ be the set of constants that occur in φ that are not members of C. Let φ^C be the result of substituting an occurrence of $f_\varphi(\alpha)$ for every occurrence of α in φ for all α in $\Sigma(\varphi, C)$. Let φ^C_λ be $\ulcorner(\lambda f_\varphi(\alpha_1)\dots f_\varphi(\alpha_n).\varphi^C)\urcorner$, where $\langle\alpha_1, \dots, \alpha_n\rangle$ is some canonical ordering of $\Sigma(\varphi, C)$. Finally, let $\vdash_C \varphi$ abbreviate the claim that $\ulcorner\top\varphi^C_\lambda\urcorner$ is true. I propose this notion as an account of what it is for a sentence φ to be valid when C is the set of expressions being treated as logical constants.

As discussed in relation to entailment in the previous section, this is only a sensible notion of validity if reality is sufficiently coarse grained for claims of tautologousness to have a chance of being true. (Both Booleanism and my theory of aboutness make reality sufficiently coarse grained; the aforementioned view of Dorr's does not, but the general strategy is sensible in his setting so long as the notion of identification is replaced with the notion of logical equivalence mentioned in footnote 7.) To see why it is a natural proposal about validity, it helps to contrast it with the Tarskian notion of validity advocated by Williamson (2003, 2013, forthcoming). φ is Tarski-Williamson valid relative to a choice of constants C just in case $\ulcorner\forall f_\varphi(\alpha_1)\dots\forall f_\varphi(\alpha_n)\varphi^C\urcorner$ is true. In other words, φ is true *however* we interpret its non-logical constants, in the sense that the result of generalizing on those constants yields a true sentence. By contrast, my proposal counts a sentence as valid just in case it is tautologous when we *leave uninterpreted* all of its non-logical constants, in the sense that the result of abstracting on those constants yields a predicate that expresses a tautologous condition.

Now, I don't want to fight about the word 'validity' – as used by philosophers it is jargon, and we should be looking to engineer a useful notion rather than attempting to analyze a purportedly pretheoretical one. Having said that, I do think there is something suggestive about the cases where my proposal issues different verdicts from the Tarski-Williamson one. To take the simplest example, in cases where every constant occurring in the sentence in question is treated as logical, the Tarski-Williamson proposal collapses validity to truth, whereas my proposal deems valid only those sentences that express tautologies. My proposal also implies that validities are closed under RN whenever $\Box$ is included as a logical constant, given the natural assumption that $\top(\lambda v_1 \dots v_n.\Box\varphi)$ whenever $\top(\lambda v_1 \dots v_n.\varphi)$. Further discussion of these issues will have to wait for another occasion.

The main advantage of my proposal for present purposes is that, since validity is not defined quantificationally, it allows us to investigate questions about the logic of quantifiers without begging the questions at issue. In particular, it allows us to relate ADJUNCTION to classical quantification theory, as codified by the validity of all instances of ∃I and the closure of validities under GEN*. I will now show how, given Booleanism (for concreteness), these claims about validity follow from ADJUNCTION. (Actually, I will only show this for sentences in which the constant being generalized on is the only non-logical constant in the sentence; in order to establish the more general result we would need the following more general adjunction principle: $(\lambda v_1 \dots v_n.\varphi) \leq (\lambda v_1 \dots v_n.\forall x\psi) \leftrightarrow (\lambda v_1 \dots v_n x.\varphi) \leq (\lambda v_1 \dots v_n x.\psi)$, where x is not free in φ.)

∃I: Suppose $\{\neg, \rightarrow, \forall\} \subseteq C$ and $\Sigma(\varphi, C) = \{a\}$. Let F be the property expressed by $\ulcorner(\lambda x.\varphi[x/a])\urcorner$. We aim to show that $\ulcorner\top(\varphi \rightarrow \exists x\varphi[x/a])^C_\lambda\urcorner$, i.e. $\ulcorner\top(\lambda x.\varphi[x/a] \rightarrow \neg\forall x\neg\varphi[x/a])\urcorner$, is true. For the reasons given at the end of section 4 about the interpretation of predicate abstraction, it suffices to show that $\ulcorner\top(\lambda x.(\lambda x.\varphi[x/a])x \rightarrow \neg\forall x\neg(\lambda x.\varphi[x/a])x)\urcorner$ is true. Assuming a disquotational theory of meaning for $\neg, \rightarrow, \forall$, and $\top$, this amounts to the claim that $\top(\lambda x.Fx \rightarrow \neg\forall x\neg Fx)$, which, given Booleanism, is equivalent to the claim that $(\lambda x.\forall x\neg Fx) \leq (\lambda x.\neg Fx)$. And this claim follows from $\forall x\neg Fx \leq \forall x(\lambda x.\neg Fx)x$ (another consequence of Booleanism and the aforementioned principle about predicate abstraction) given the left-to-right direction of ADJUNCTION. So $\vdash_C \varphi \rightarrow \exists x\varphi[x/a]$.

GEN*: As above, except we drop the assumption that C includes $\neg$ and $\rightarrow$. Suppose $\vdash_C \varphi$. So $\ulcorner\top\varphi^C_\lambda\urcorner$, i.e. $\ulcorner\top(\lambda x.\varphi[x/a])\urcorner$, is true. Given our disquotational assumptions, this is equivalent to the claim that $\top F$, which, given Booleanism, is equivalent to the claim that $(\lambda x.\forall xFx \rightarrow \forall xFx) \leq F$. By the right-to-left direction of ADJUNCTION, we have $(\forall xFx \rightarrow \forall xFx) \leq \forall xFx$, which given Booleanism implies $\top\forall xFx$. Given our disquotational assumptions, it follows that $\ulcorner \top\forall x(\lambda x.\varphi[x/a])x \urcorner$ is true, and hence so too is $\ulcorner \top\forall x\varphi[x/a] \urcorner$, i.e. $\ulcorner \top\forall x\varphi[x/a]^C_\lambda \urcorner$, again for the reasons given in section 4. So $\vdash_C \forall x\varphi[x/a]$. By conditional proof, if $\vdash_C \varphi$, then $\vdash_C \forall x\varphi[x/a]$.

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