A Theory of Aboutness

Jeremy Goodman (USC)

The theory is formulated in a simply relationally typed higher-order language, in which e is a type, every finite sequence of types is a type, and nothing else is a type. Type indications are suppressed using the convention that lowercase "x"s are always of type e and all other letters are 'typically ambiguous', so that any uniform substitution of closed terms for schematic letters yielding a well-formed formula counts as an instance of a schema.

1 Decomposition

$y = z := \forall X(Xy \leftrightarrow Xz)$

UNIQUE QUALITATIVE DECOMPOSITION
$(\mathcal{Q}R \wedge \mathcal{Q}S \wedge Rx_1 \dots x_n = Sx_1 \dots x_n \wedge \bigwedge_{i \neq j} x_i \neq x_j) \rightarrow R = S$

2 Aboutness

$\mathcal{A}(p, x) := \exists F(p = Fx)$; $\mathcal{A}(F, x) := \exists R(F = (\lambda y.Ryx))$; *etc.*

QUALITATIVENESS *qua* ABOUTNESSLESSNESS
$\mathcal{Q}X \leftrightarrow \neg\exists x\mathcal{A}(X, x)$

Consequences:

i) $\mathcal{Q}p \rightarrow p \neq (\lambda x.p)x$

ii) $\mathcal{Q}p \rightarrow p \neq (p \wedge (Fx \vee \neg Fx))$

UNIQUE EXTRACTABILITY
$\mathcal{A}(X, x) \rightarrow \exists! Y(\neg\mathcal{A}(Y, x) \wedge X = (\lambda y_1 \dots y_n.Yy_1 \dots y_n x))$

ATOMIC ABOUTNESS
$\mathcal{A}(Xy_1 \dots y_n, x) \rightarrow \mathcal{A}(X, x) \vee \mathcal{A}(y_1, x) \vee \dots \vee \mathcal{A}(y_n, x)$

where $\mathcal{A}(x, y) := x = y$

3 Coarseness

NON-VACUOUS-β
$(\lambda y.\varphi)a = \varphi[a/y]$, provided y occurs free in φ

Consequences:

i) $(\lambda x.y = x)y = (\lambda x.x = x)y$

ii) $(\lambda xy.x = x \wedge y = y)zz = (\lambda xy.x = y \wedge x = y)zz$

iii) $(\lambda X.X = X)(\lambda p.p) = (\lambda X.X(X = X))(\lambda p.p)$

iv) $X = (\lambda yz.Xzy) \rightarrow (\lambda Y.Y = Y)X = (\lambda Y.Y = (\lambda yz.Yzy))X$

$p \equiv_L q := p \wedge q = p \vee q$

$p \equiv_N q := p \vee \neg p = q \vee \neg q$

TWO-DIMENSIONALISM
$p \equiv_L q \wedge p \equiv_N q \rightarrow p = q$

BOOLEAN LOGICAL CONTENT
$\varphi \equiv_L \psi$, whenever $\varphi \leftrightarrow \psi$ is a theorem of propositional logic

OBJECTUAL NON-LOGICAL CONTENT
$p \equiv_N q \leftrightarrow \forall x(\mathcal{A}(p, x) \leftrightarrow \mathcal{A}(q, x))$

4 Modalities

$\Box p := p = (p \rightarrow p)$

$\Box\varphi$, whenever φ is a theorem of classical higher-order logic

We have the necessity of identity, so the logic of $\Box$ includes S4; but it is weaker than S5, because the theory proves:

THE POSSIBILITY OF IDENTITY[1]
$\Diamond(x = y)$

which given the existence of at least two individuals (which is consistent in the theory) is inconsistent with the necessity of distinctness, which follows from the necessity of identity in S5.

[1] *Proof*: Obvious if $x = y$, so assume $x \neq y$. $\mathcal{Q}(\lambda xy.\neg(x = y))$ and $\mathcal{Q}(\lambda xy.\neg(x = y) \rightarrow \neg(x = y))$. But $(\lambda xy.\neg(x = y)) \neq (\lambda xy.\neg(x = y) \rightarrow \neg(x = y))$. So by UNIQUE QUALITATIVE DECOMPOSITION, $(\lambda xy.\neg(x = y))xy \neq (\lambda xy.\neg(x = y) \rightarrow \neg(x = y))xy$; hence $\neg\Box\neg(x = y)$, by NON-VACUOUS-β.